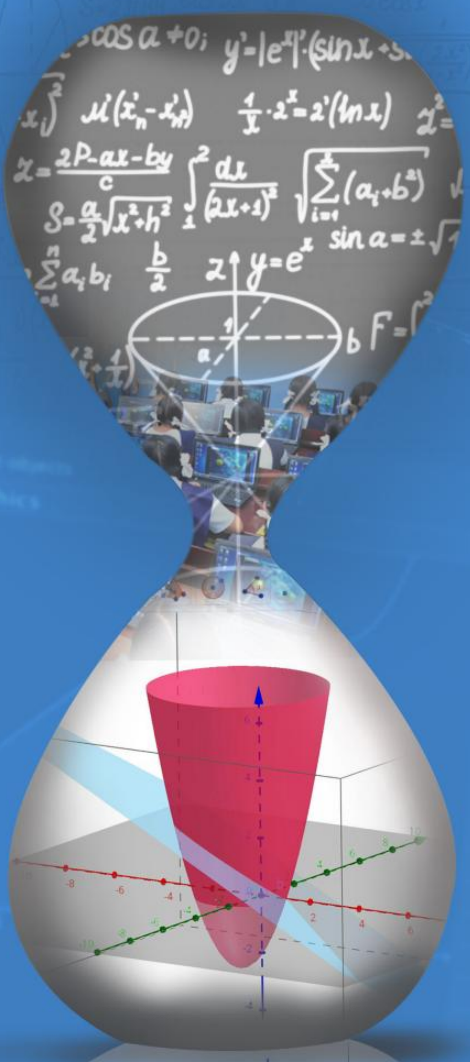
Higher Secondary

IT MATHS LAB MANUAL

FIRST YEAR



State Council of Educational Research and Training (SCERT)
Vidyabhavan, Poojappura, Thiruvananthapuram
Kerala - 695012. Website: www.scert.kerala.gov.in
e-mail : scertkerala@gmail.com

Government of Kerala
Department of General Education

Higher Secondary

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2019

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Poojappura PO
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Foreword

The field of general education in Kerala is undergoing drastic changes by incorporating innovative methods in the teaching learning process. The different initiatives that are being implemented after scientific research and planning, take into consideration the impact of such changes in the academic excellence of the learner community. Such a new wave of thought has been bringing in many a change in the teaching of Mathematics in Kerala. At the primary level, Mathematics may be of concrete concepts, which, during its gradation to the Higher Secondary level, becomes a greater universe of abstraction which requires proper visualisation. Learners get the opportunity to learn Mathematics using free software like GeoGebra up to Standard X. But at the Higher Secondary level, its continuity is lost to a certain extent as there is no scope for IT enabled learning in Mathematics.

Considering this, the SCERT Kerala is introducing the concept of IT Maths Lab for Higher Secondary students from the academic year 2019-20 onwards. This IT Maths Lab Manual will surely help all children as a source of inspiration in exploring Mathematics and support all learners to attain the concepts meaningfully. With regards,

Dr.J.Prasad,

Director,

SCERT

,

The Team

Development Team

1. Achuthan C G, GHSS Karakkurissi, Palakkad
2. Dr K Shanavas, Master Trainer, KITE, Malappuram
3. Jayadas T, TRKHSS Vaniyamkulam, Palakkad
4. M A Rafi, AMHSS Poovambayi, Kozhikode
5. Pramod M K, GMMGHSS Palakkad
6. R Ramanujam, MNKM Govt.HSS Pulappatta, Palakkad
7. Sajeev C S, Govt.Girls VHSS, Manacaud, Thiruvananthapuram
8. Sajith K, Nochat HSS, Kozhikode
9. Sudeep P, Dr K B Menon Memorial HSS, Thrithala, Palakkad
10. Vinod Kumar A, PKMMHSS, Edarikode, Malappuram
11. Vinod Kumar K, GVHSS Krishnapuram , Alappuzha

Experts

1. Dr. E Krishnan, Retd Professor, University College, Thiruvanthaparam
2. Prof. Sukesh A, Govt. College of Engineering, Kannur

Programme Coordinator:
Dr. K S Sivakumar
SCERT Thiruvananthapuram

Preface

IT Maths Lab is introduced in Higher Secondary level to make the learning process more creative and child friendly. Maths Lab opens a platform for *learning by doing* and it accelerates learning by making it joyous. Mathematics skills are inherent in a child and it blossoms through schooling. Children acquire knowledge of numbers and numerical skills in their early childhood itself through plays and activities. The way of presentation of mathematical concepts becomes a bit abstract in higher classes and it spoils the genuine interest of children in the subject. For example, we attain the concept of area of a rectangle or volume of a rectangular prism through appropriate activities. But it becomes a herculean task to find an activity suitable for the class room, to impart the idea of maximum surface area of a rectangular prism whose volume is fixed. If a learning process demands tedious sequence of logical arguments, then it would be, better if we make it an activity and let the students do it and learn the concept.

Fortunately it has become easy today to makeover a concept to an activity by the use of technology and kids are specially equipped in keeping the pace with it. Software like GeoGebra contribute a lot in this makeover. Activities in IT Maths Lab are regulated with the help of GeoGebra. A detailed Lab Manual is also set for helping the students.

- For the first year, there are 16 labs from the class 11 syllabus , out of which 8 are to be done. Those who are interested can go through all the labs.
- Each lab consists of 3 or 4 activities and some additional activities.
- Additional activities are optional.
- Additional activities are provided almost in all labs, meant for only those who are interested.
- To familiarise the tools of GeoGebra,the lab 'Basic Concepts' is included in the manual. It gives a brief description of the basic tools of GeoGebra.
- Manual is so set that students can independently do lab using the it.
- Step by step explanation is given for each activity.
- Separate markings are given for those steps which are to be written in the observation book.
- Each student has to keep an observation book, which is to be examined and rectified by the teacher.

Our class room environment limits the scope of innovative learning. Hopefully this will be well overcome by the IT Maths Lab

Contents

Basic Concepts

Aim

- To familiarise the GeoGebra interface and Toolbar
- To familiarise the concept of domain, range and graphs of standard functions

Concepts

- Domain, range and graphs of functions

Discussion

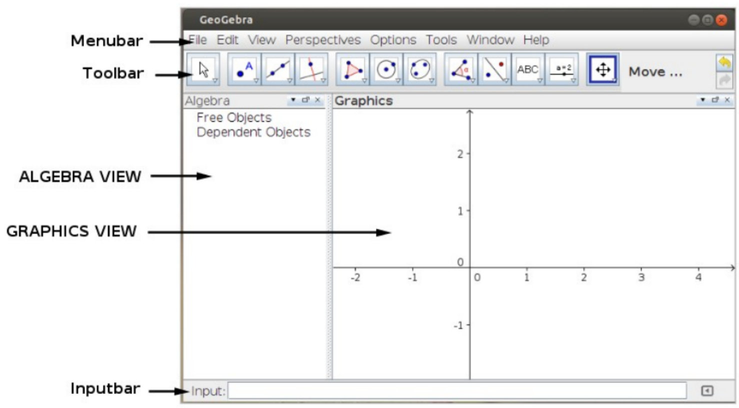
Many of us are already familiar with the software GeoGebra which leads us to the joy of dynamism of Geometry. In Higher Secondary Mathematics, we deal with concepts like Analytic Geometry, Trigonometry, Calculus etc. in which GeoGebra can contribute a lot in conceptual understanding.

In this lab, we learn some basic tools and commands of GeoGebra which will help us in our learning process. We also learn about input commands to plot the graphs of polynomial functions.

Activity 0.1 GeoGebra Interface

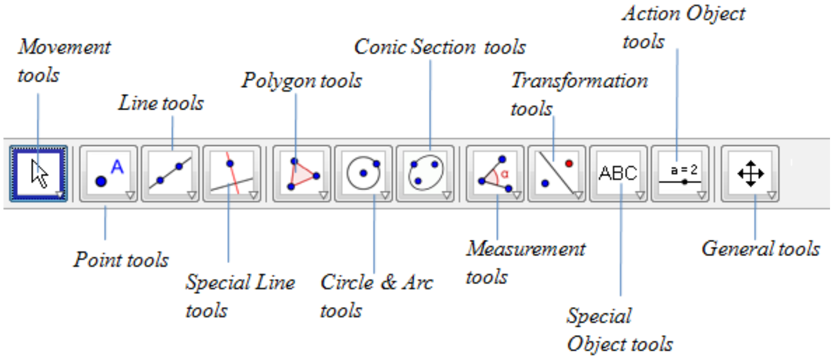
Procedure:

- Familiarise the interfaces of GeoGebra

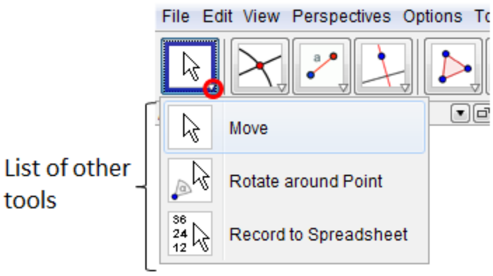


We can draw geometrical figures or graphs in the Graphics View by selecting tools from the Toolbar or by giving commands in the Input Bar.The algebraic form of the figures or graphs drawn in the Graphics View is available in the Algebra View. Apart from the Graphics View and Algebra View, GeoGebra also offers Graphics 2, Spreadsheet, CAS (Computer Algebra System) and 3D Graphics. All these views can be shown or hidden using "View" menu.

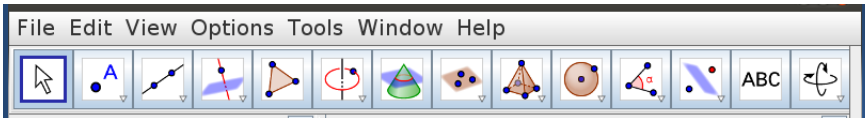
- Familiarise the Toolbar and some important tools of GeoGebra
  In GeoGebra Graphics View, tools are arranged in 12 sets as shown in the figure below.



All the tools in each set are obtained by clicking on the small arrow at the bottom right corner of each icon as shown in figure. Keeping the cursor on the tool, a brief description of the function of the tool is displayed.



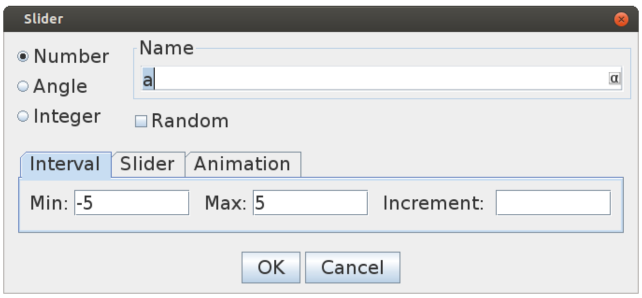
When the 3D Graphics View is enabled, the tools will change accordingly.



Activity 0.2 Graph of a Function

Procedure:

- Create a slider **a** with increment 1 as follows
  Using `Slider` tool click anywhere on the **Graphics** View. We get a window in which we can edit the name, minimum value, maximum value, and increment of the slider.



- Plot the point $A(a, a^2)$. (Input: `A=(a,a^2)` or `(a,a^2)`)
- Change the value of the slider and observe the movement of A.

  We can change the value of a slider in different ways.

  – Click and drag the slider point.
  – Using `Move` tool click on the slider point and then use arrow keys to change the value.
  – Right click on the slider and select **Animation On**
  – Create an input box for the slider and change the value
- Change the increment of the slider to 0.01.(Right click on the slider → Object Properties → Slider → Enter 0.01 in the **Increment** box.)
- Observe the movement of the point.
- Trace the point A. (Right click on the point→ Trace On)

  Observe the curve traced. What does it represent ?

Input Box

- To create an Input Box for the slider **a**, using the **Input Box** tool, click anywhere on the Graphics View. We get the 'Input Box' window. Enter any caption (say Slider). From the **Linked Object** drop down menu select **a**.
- To create an Input Box for the point A, from the **Linked Object** drop down menu select **A**.

Locus Tool

- We can draw the path traced by the point A using **Locus** tool. For this, using the tool click on the slider **a** and on the point A.

- Create an Input Box for the point A.
- Change the definition of the point A as (a, a^3). (In the Input Box enter `(a,a^3)`)

Observe the curve traced. What does it represent ?

What should be the definition of the point A, so that the curve represents the graph of the function $f(x) = x^4$?

Activity 0.3 Standard Functions

Procedure:

- Draw the graphs of standard functions using inputs

Function	Input
x	`x`
x^2	`x^2`
$\|x\|$	`abs(x)`
$\sqrt{x}$	`sqrt(x)`
x^3	`x^3`
$[x]$	`floor(x)`
$\frac{1}{x}$	`1/x`
Signum function	`sign(x)`

– We can see the name of each function in the Algebra View

– By clicking the bullets, we can hide/show the graph of the function

Observe the graph of each function and find its domain and range

- Input: `ceil(x)`.

Observe the graph of the function obtained and compare this with the graph of the floor function. Define this function

Activity 0.4 Domain and Range

Procedure:

- Create an integer slider **n**
  (Using `Slider` tool click anywhere on the Graphics → select Integer → click OK. If we want we can change the minimum , maximum and increment of the slider.)

- Draw the graph of $f(x) = x^n$
  [Input: `f(x) = x^n`]

  Observe the graph of the function x^n and find the domain and range for different values of n

  What happens to the graph of the function x^n between -1 and 1 as n becomes larger and larger ? why ?

n	Function	Domain	Range
1	x		
2	x^2		
3	x^3		

Additional Activities

Activity 0.A Polynomial Function

Discussion:

We discuss how the domain and range of a polynomial function related to its degree.

Procedure:

- Draw the graphs of some polynomial functions.(eg. for getting the graph of $f(x) = x^3 + 2x^2 - 3$, Input: `f(x)=x^3+2x^2-3`)

  Draw the graphs of the following functions and find their domain and range

Sl No	Function	Domain	Range
1	$2x^3 - 3x + 4$		
2	$-x^2 + 2x - 3$		
3	$3x^4 + 5$		

  What is your inference about the domain and range of polynomial functions?

Activity 0.B Functions With Rational Powers

Discussion:

We discuss the nature of the function $f(x) = x^{\frac{1}{n}}$ for integer values of **n**

Procedure:

- Create an integer slider **n** (min=1, max=10)

- Draw the graph of $f(x) = x^{\frac{1}{n}}$
  (Input: `f(x)=x^(1/n)`)

Move the slider and observe the graph. Identify the change in domain, range and the graph when **n** takes even and odd values.

Also draw the graph of x^n and compare it with the graph of $x^{\frac{1}{n}}$.

Lab 1

Value of Functions

Aim

- To construct an applet to establish geometrically the correspondence of a number and its image under a function.
- To use this applet to find the images of numbers under various functions.
- To use an applet to visualise the comparison of a function with an input-output machine.

Concepts

- Image of a number a under a function f is denoted by $f(a)$
- Graph of the function f is the collection of points $(a, f(a))$

Discussion

For any number a, the ordered pair $(a, f(a))$ is a point on the graph of the function f, so its y coordinate gives the value of $f(a)$. We use this idea for constructing our applet. Once such an applet is constructed, we can simply change the function and use it for different functions.
Sometimes we compare a function with a machine which gives an output, according to the definition of the function, for a given input. In Activity 1.3 we use an applet which helps us to visualise this comparison. By this activity we get a clear idea about the domain of the function.

Activity 1.1 Functions

Procedure:

- Draw the graph of $f(x) = x^2$.
- Create a number slider **a** with increment 0.01
- Plot the points $A(a, 0)$, $B(a, f(a))$, $C(0, f(a))$. (Give inputs like `A=(a,0)`).
- Draw the line segments AB and BC using `Segment` tool.
- Show the coordinates of A, B, and C.
- Now drag the point A along the x axis (either click and drag the point or using slider - click and drag the slider point to change the value of **a**) and observe the movement of C on the y axis.

> To show the coordinates of a point, right click on the point. Go to Object Properties → Basic → Show Label and select the Name and Value option

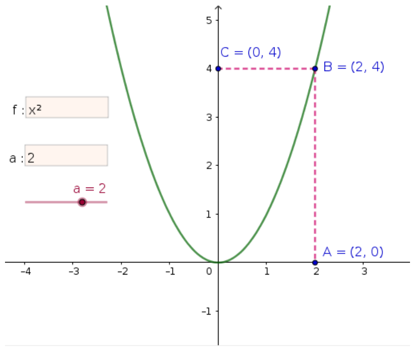
Using this, find the values of $(2.3)^2$, $(-1.8)^2$, $(0.9)^2$, $(2.9)^2$ ...

Save the file as Activity 1.1

Activity 1.2 Values of Functions

Procedure:

- Open the file Activity 1.1 and save as Activity 1.2
- Create an Input Box for f and change the function using it.
  (Select `Input Box` tool, $\rightarrow$ Click on `Graphics View` $\rightarrow$ give a suitable caption (say function) $\rightarrow$ `Linked Object` $\rightarrow f(x) = x^2 \rightarrow$ OK)
- Similarly create an Input Box for the slider.



Change the functions accordingly and find the approximate values corrected to 3 decimal places of the following

	$3^{\frac{1}{3}}$	$\sqrt{1.8}$	$2^{\frac{2}{3}}$	$\sqrt{\sqrt{5}}$	$(3.46)^{\frac{-3}{2}}$
Function	$x^{\frac{1}{3}}$				
Input(x)	3				
Value($f(x)$)					

We can set the number of decimal places as follows;
Options $\rightarrow$ Rounding $\rightarrow$ Select number of decimal places.

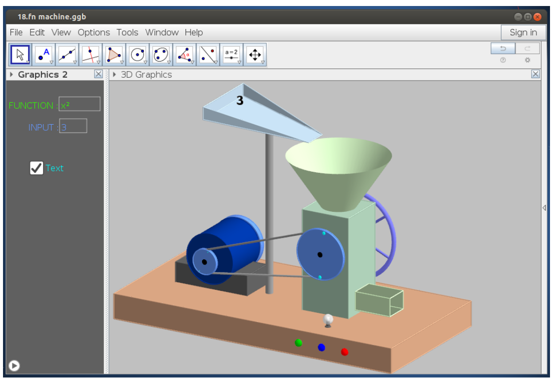
Change the function to $f(x) = \frac{1}{x}$, and observe how the point C moves as the point A approaches the origin from either side.

Change the function to $f(x) = [x]$ and observe the movement of C according to A

Activity 1.3 Function Machine

Procedure :

Use Applet ML 1.3



About the Applet

Three switches are provided on the machine

- GREEN :- Click to start the machine.
- RED :- Click to stop the machine.
- BLUE :- Click to reset.

Using Input Boxes we can change the function and the input number.
The warning light provided on the machine turns red if the input number is out of the domain of the function.

Change the function to $f(x) = \sqrt{x}$ and find the values of the following.

i) $\sqrt{2}$ ii) $\sqrt{1.8}$ iii) $\sqrt{\frac{2}{3}}$

What happens if we give a negative number as the input ?

Change the function to $f(x) = \frac{1}{x}$ and find the values of the following.

i) $\frac{2}{3}$ ii) $\frac{-3}{7}$ iii) $\sqrt{\frac{2}{3}}$

What happens if the input is 0 ?

Additional Activities

Activity 1.A Temperature Scales

Discussion:

There are various scales to measure temperature. Perhaps the most popular ones are the Fahrenheit and the Celsius scales.
$F(C)$ is the Fahrenheit temperature corresponding to the Celsius temperature C and they related to each other as

$$F(C) = \frac{9}{5}C + 32$$

- Plot the graph of the above function (Consider C as the variable x)

  From the graph identify the Celsius temperature at which the Fahrenheit temperature become zero

  From the graph identify the Fahrenheit temperature at which the Celsius temperature become zero

While plotting the graph of $F(C)$ we have to use x instead of C. So in order to get the graph input **9x/5+32**

Lab 2

Shifting of Graphs

Aim

To analyse the changes in the graph of a function according to some slight changes in the definition

Concepts

- Graph of a function

Discussion

If we know the graph of the function $f(x)$ we can obtain the graphs of the functions $f(x)+a$, $f(x+a)$, $-f(x)$ and $f(-x)$ by translation or reflection.

This idea helps us to imagine the graphs of some functions if the graph of the base function is known.

This activity gives insight on the concept of a family of curves.

Activity 2.1 Shifting of graphs : $f(x)+a$

Procedure:

- Draw the graph of $f(x) = x^2$
- Create a number slider **a** with increment 0.1
- Draw the graph of $g(x) = f(x) + a$ (Input: `f+a`)

  Observe how the graph of $g(x)$ changes according to **a**
- Create Input Boxes for editing function and slider **a**

  Do the above observations for different functions such as $|x|$, $[x]$, x^3 etc.
- Save this as Activity 2.1

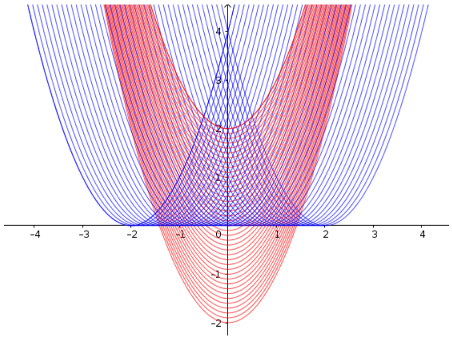
Apply trace to the graph to get a pattern (rightclick → trace on). To erase the pattern, press Ctrl+F

Change the value of slider by these methods :

- Click on the slider point and move
- Using *Move* tool, select the slider and use arrow keys
- Right click on the slider and turn on animation

Activity 2.2 Shifting of graphs : $f(x+a)$

Procedure:



- Open a new GeoGebra window.
- Draw the graph of $f(x) = x^2$.
- Create a number slider **a** with increment 0.1
- Draw the graph of $g(x) = f(x+a)$. (Input: `f(x+a)`)

  Observe how the graph of $g(x)$ changes according to **a**.

- Create Input Boxes for the function f and slider **a**.

  Generalise the above observations with different functions such as $|x|$, $[x]$, x^3 etc.

- You may use the animation option to change the slider.
- Save this as Activity 2.2

Activity 2.3 Reflection of a graph : $-f(x)$

Procedure:

- Open a new GeoGebra window.
- Draw the graph of $f(x) = x^2$
- Draw the graph of $g(x) = -f(x)$ (Input: `-f`)

  Compare the graphs of $f(x)$ and $g(x)$.

- Create an Input Box for f and change the function to

  i) $x^2 + 2$ ii) $x^2 - 1$ iii) $|x| - 1$ iv) $|x - 1|$

  v) $[x]$ vi) $x^2 + 2x + 1$ vii) $\dfrac{1}{x}$

  Compare the graphs of f and g in each case. Write your findings.

- Save this file as Activity 2.3

Activity 2.4 Reflection of a graph : $f(-x)$

Procedure:

- Open a new GeoGebra window
- Draw the graph of $f(x) = x^3$
- Draw the graph of $g(x) = f(-x)$ (Input: `f(-x)`)

  Compare the graphs of $f(x)$ and $g(x)$

- Create an Input Box for f and change the function to

  i) $\frac{1}{x}$ ii) $[x]$ iii) $|x|$

  iv) x^2 v) $(x-2)^2$

  Compare the graphs of f and g in each case. Write your findings.

What is the speciality of the graphs of odd and even functions?

Identify odd and even functions discussed in this lab.

Is there any function which is neither odd nor even?

> A function $f(x)$ is an even function if $f(-x) = f(x)$ and an odd function if $f(-x) = -f(x)$.

- Save this file as Activity 2.4

Additional Activities

Activity 2.A Translations of graphs: 1

Procedure:

- Draw the graph of $f(x) = x^2$.
- Create a number slider **a**, with increment 0.1
- Draw the graph of $g(x) = f(x-a) + a$.

  Observe how the graph of $g(x)$ changes according to **a**.

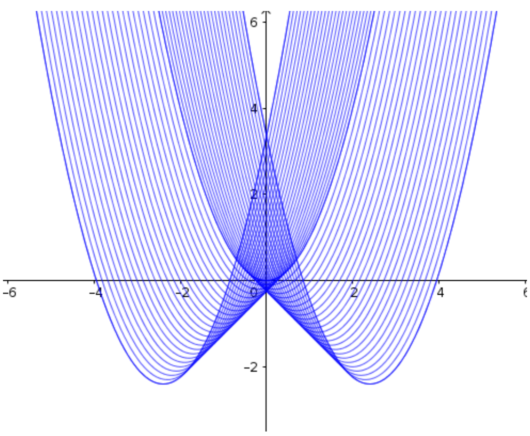
- Create an Input Box for g and change the function to

  i) $f(x-a) - a$ ii) $f(x-a) + 2a$

  iii) $f(x-a) + 3a$ iv) $f(x-a) - 3a$

  Observe the shift in the graph of g according to the change in **a**.

  Try to draw the pattern given in the figure.



Activity 2.B Translations of graphs: 2

- Draw the graph of $f(x) = x^2$.
- Create number sliders **a** and **b** , with increment 0.1
- Draw the graph of $g(x) = f(x + a) + b$.

- By adjusting the values of a and b transform the graph of x^2 to that of the following functions.

  i) $(x + 2)^2 - 3$ ii) $x^2 + 6x + 9$ iii) $x^2 - 4x + 6$

Activity 2.C Family of curves - using sequence command

Using sequence command, we can represent the family of curves obtained by shifting a graph

Procedure:

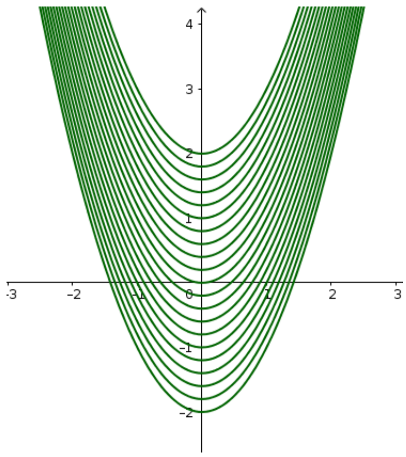
- Draw the graph of $f(x) = x^2$
- In the Input Bar, give the command, `Sequence[f+i,i,-3,3,0.2]`, which gives the graphs of the functions

  $x^2 - 3,\ x^2 - 2.8,\ x^2 - 2.6,\ \ldots,\ x^2,\ \ldots,\ x^2 + 3$

In the Input Command Sequence[f+i,i,-3,3,0.2], f is function, i is variable, -3 is start value, 3 is end value and 0.2 is increment

Imagine the family of curves obtained by the following input commands and then draw them.

1. `Sequence[f(x+i),i,-3,3,0.2]`
2. `Sequence[f(x-i)+i,i,-3,3,0.2]`
3. `Sequence[f(x-i)-i,i,-3,3,0.2]`
4. `Sequence[f(x-i)+2i,i,-3,3,0.2]`



Create a slider **a** and input the command `Sequence[f(x-i)+a*i,i,-3,3,0.2]`. Observe the pattern for different values of **a**.

Create an Input Box for f and observe the pattern for different functions

Lab 3

Domain and Range

Aim

- To find the domain and range of functions from their graphs

Concepts

- Graph of a function, domain and range, shifting of the graph

Discussion

A graph, being the pictorial representation of a function, gives much information about the properties of that function.

In this lab, we discuss the domain and range of functions with the help of their graphs. We also discuss rational functions, functions with restricted domain and piecewise functions.

In each problem we discuss here, first try to imagine the graph, domain and range of the function and then draw it using GeoGebra

Activity 3.1 Domain and Range of Functions from their Graphs

Procedure:

Imagine the graphs of the following functions and write their domain and range. (You may use the idea of shifting and reflection of graph as in Lab 2)

Check your answer by drawing the graphs using GeoGebra

i) $x^2 + 2$ ii) $x^2 - 3$ iii) $3 - |x|$ iv) $(x + 2)^2 - 1$

v) $x^2 - 6x + 12$ vi) $|x - 2|$ vii) $|x - 2| + 3$ viii) $2x^2 - 8x + 5$

ix) $\frac{1}{2}[x]$ x) $[\frac{x}{2}]$ xi) $x - [x]$ xii) $3 - x^2$

xiii) $\sqrt{x - 2}$ xiv) $\sqrt{4 - x}$ xv) $\dfrac{1}{x - 2}$ xvi) $\sqrt{x^2 - 4}$

xvii) $\sqrt{9 - x^2}$ xviii) $\dfrac{1}{x^2 - 9}$ xix) $\dfrac{x^2}{x^2 + 1}$

Activity 3.2 Rational Functions

Procedure:

- Draw the graph of the function $\frac{x^2-4}{x-2}$. What is the domain of this function?
- On Graphics 2, draw the graph of the function $g(x) = x + 2$

> To open Graphics 2 go to View → Graphics 2

Observe the graphs of f and g. Are they one and the same? Does it mean that $f = g$

- Create a slider **a**
- In Graphics View, plot the point $A(a, f(a))$ and in Graphics 2 plot $B(a, g(a))$
- Change the value of **a**. We can see that the points move along the graphs.

What happens to the points when **a** reaches 2 ? What do we infer? comment on it.

Activity 3.3 Piecewise Functions

Procedure:

- We can draw the graphs of functions with restricted domains using `If` command.
  For example
  `If[-1<=x<=2,x^2]` gives the function $f(x) = x^2$ in $[-1, 2]$

  (We can also use the `Function` commad. `Function[x^2,-1,2]`) gives the same function.

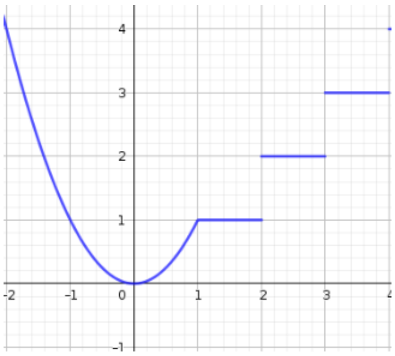
  `If[x<=2,x^2,x>2,2x]` gives the function
  $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 2x & \text{if } x > 2 \end{cases}$

> The If command in GeoGebra has the following
>
> If[c,f]
>
> gives the function f only for the values of x satisfying the condition c.
>
> If[c,f,g]
>
> gives the function f for the values of x satisfying the condition c and g for all other values of x.

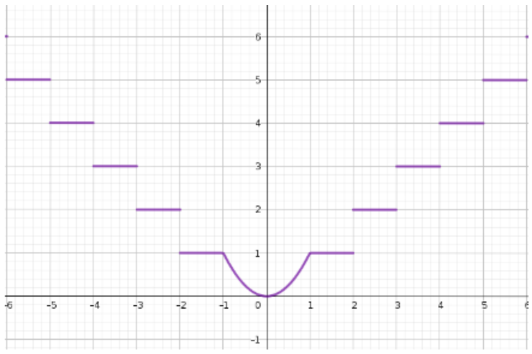
Observe the graphs of the following functions and find their domain and range

1. $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 2x+1 & \text{if } x > 2 \end{cases}$
2. $f(x) = \begin{cases} x^3 & \text{if } x \leq 0 \\ x^2+1 & \text{if } x > 0 \end{cases}$
3. $f(x) = \begin{cases} x^2+2 & \text{if } x < 0 \\ -x^2-2 & \text{if } x > 0 \end{cases}$
4. $f(x) = x^2$ in $[-2, 1]$
5. $f(x) = x^3$ in $[-2, 2]$
6. $f(x) = \frac{1}{x}$ in $[-1, 2]$

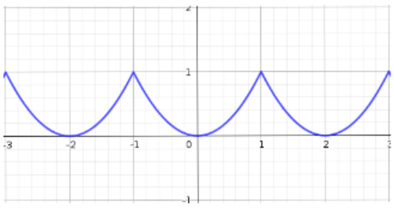
Identify the functions and try to draw the graphs given in the figures.



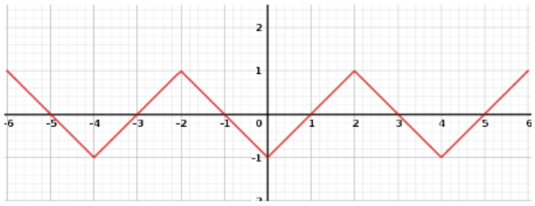
(a)



(b)



(c)



(d)

Additional Activities

Activity 3.A Leaking Tank

A tank holds 50 gallons of water. There is a leak at the bottom of the tank through which water is draining out.

If it takes 20 minutes to completely drain out the water, the volume of water in gallons remaining in the tank at any time t (minutes) is given by Toricelli's law as

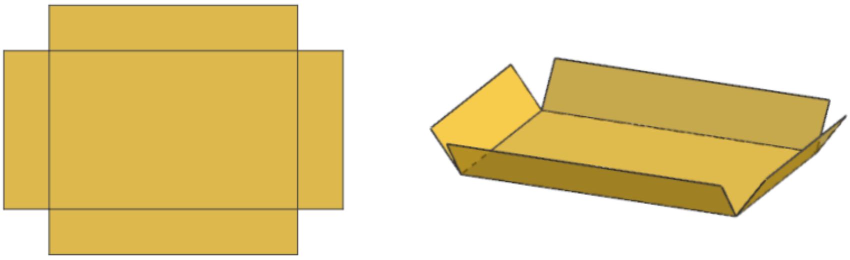
$$V(t) = 50\left(1 - \frac{t}{20}\right)^2 \qquad 0 \leq t \leq 20$$

Find the volume of water in the tank at times 0,5,10,15 and 20 minutes

What is the domain and range of this function ?

Activity 3.B The Volume of a Box

An open box is to be made from a cardboard of size 5 m by 3 m by cutting out squares of equal size from all the four corners of the cardboard and folding up the sides as shown in the figure below.



- let x represents the length of the square cut away from the cardboard

  Write the volume of the box as a function of x

- Plot the graph of this function

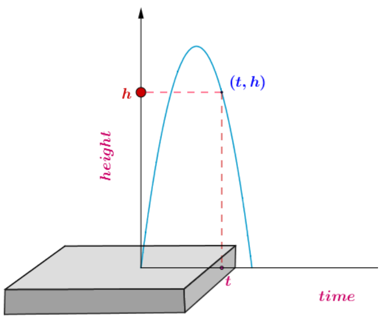
  What is the domain of this function ?

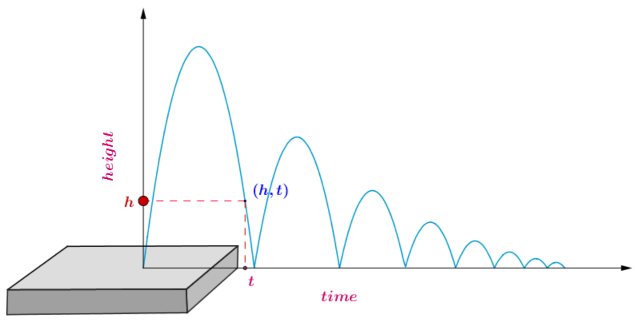
  What is the range of this function ?

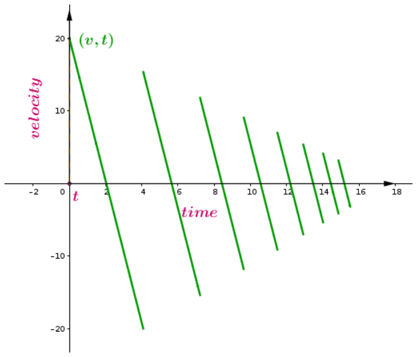
  Find the maximum volume of the box. What should be the side of the square to be cut away to get the box of maximum volume ?

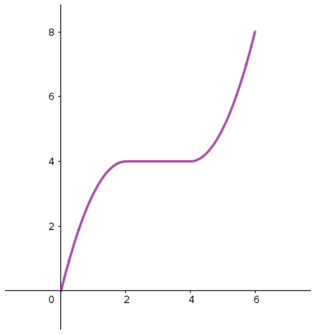
Activity 3.C Some Familiar Graphs from Physics

Observe the following graphs, describe the physical situations involved in the depiction of the graphs. Try to draw the graphs.









Activity 3.D Domain and Range of Relations

Procedure:

Guess the domain and range of the following relations.

1. $R_1 = \{(x, y) : x, y \in \mathbb{R},\ x^2 + y^2 = 4\}$ (Input: `x^2+y^2=4`)
2. $R_2 = \{(x, y) : x, y \in \mathbb{R},\ x^2 + y^2 \leq 4\}$ (Input: `x^2+y^2<=4`)
3. $R_3 = \{(x, y) : x, y \in \mathbb{R},\ x^2 + y^2 \geq 4\}$ (Input: `x^2+y^2>=4`)
4. $R_4 = \{(x, y) : y \leq x^2 + 2\}$ (Input: `y<=x^2+2`)
5. $R_5 = \{(x, y) : y \geq x^2 + 2\}$ (Input: `y>=x^2+2`)

Draw the regions represented by these relations on the set of real numbers $\mathbb{R}$. Find their domain and range and verify your answer

Lab 4

Trigonometric Functions

Aim

- To create an applet to find the values of trigonometric functions and plot their graphs
- To establish some behaviours of trigonometric functions in different quadrants

Concepts

- Concept of circular functions
- Graph of the function f is a collection of points of the form $(a, f(a))$ for all values of a in its domain

Discussion

If a point is rotated from $(1, 0)$ along the unit circle centred at the origin, by an angle x radians, the x and y coordinates of the point represent $\cos x$ and $\sin x$ respectively. We define all other trigonometric functions in terms of $\cos x$ and $\sin x$. We use this idea to construct our applet.

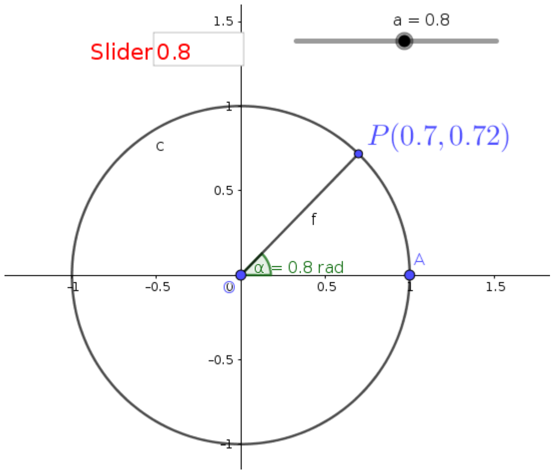
Activity 4.1 Values of Trigonometric Functions

Procedure:

Open a new GeoGebra window, do some initial settings as follows
`Options`→ `Advanced` → `Angle unit` → `Radian`

- Plot the point $O(0, 0)$ (input `O=(0,0)`)
- Draw a unit circle centred at the origin O
- Plot the point $A(1, 0)$ (input `A=(1,0)`)
- Create a number slider **a** with min = -10 , max = 10 and increment 0.01. While creating the slider, set its animation as increasing
- Plot another point A' such that $\angle AOA' = \mathbf{a}$ radian
- Rename the point A' as P (right click → Rename)
- Show the coordinates of P
- Join OP using a line segment
- Create an input box for the slider **a**.

> - To set the animation of a slider as increasing, right click on the slider and in the object properties, select increasing option from the repeat drop-down menu
> - To create an angle AOA' with measurement a, use *Angle with given Size* tool, click on A, O and then give **a** as the angle in the box



Animate the slider, observe the coordinates of the point P, hence find the domain and range of $\sin x$ and $\cos x$

Find the values of $\sin x$ and $\cos x$ for the given values of x

x	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	$\frac{\pi}{2}$	0.3	0.6	2	-1.5	-3.1	7.5
$\sin x$										
$\cos x$										

For giving π in the input box, input: *pi*

Identify the values of x for which $\sin x$ and $\cos x$ become 0, 1, -1

- Save this file as Activity 4.1

Activity 4.2 Graphs of Trigonometric Functions - 1

Procedure:

- Save file Activity 4.1 as Activity 4.2 using save as option
- Open Graphics 2 [`view` → Graphics 2]
- Plot the point $B(a, y(P))$. [$y(P)$ gives the y coordinate of P]
- Give trace to this point and animate the slider

  Observe the path of this point. What does this path represent?
- Save the file.

- We can see the path of the point using locus tool also. To get the path, take the locus tool, click on the point and on the slider
- For analysing the graphs of trigonometric functions, it is more convenient to mark $\cdots -\frac{\pi}{2}, 0, \frac{\pi}{2} \cdots$ on the x axis instead of $\cdots -1, 0, 1 \cdots$
  (For this right click on the Graphics 2. Go to *Object Properties* Change the x Axis distance to $\frac{\pi}{2}$)
- We can draw the graphs of $\sin x$ and $\cos x$ using input commands sin(x) and cos(x).

Activity 4.3 Graphs of Trigonometric Functions - 2

Procedure:

- Open Activity 4.2 and save as Activity 4.3 using save as option
- Create an input box for the point B
- Change the definition of B as $(a, x(P))$

Observe the path of this point

What does this path represent?

Redefine B as $(a, \frac{1}{y(P)})$ and $(a, \frac{y(P)}{x(P)})$, observe the path of P and identify the functions.

What should be the definition of B for getting the graphs of $\sec x$ and $\cot x$?

Observe the values of trigonometric functions, write their domain, range and complete the following table.

Function	$\left(0, \frac{\pi}{2}\right)$	$\left(\frac{\pi}{2}, \pi\right)$	$\left(\pi, \frac{3\pi}{2}\right)$	$\left(\frac{3\pi}{2}, 2\pi\right)$
$\sin x$	Positive			
	Increasing from 0 to 1			
$\cos x$				
$\tan x$				
			Increasing from 0 to ∞	
$\sec x$				
$\cot x$				
cosec x				

Additional Activities

Activity 4.A $k\sin(x)$

Discussion :

We construct an applet similar to that in Activity 4.1, using which we describe the functions $k\sin(x), k\cos(x)$ etc. for different values of **k**.

Procedure :

Do the initial settings as in Activity 4.1

- Create two sliders, **k** with Min = 0 and **a** with Min = -10 , Max = 10 and increment 0.01. While creating the slider **a**, set its animation as increasing
- Draw a circle of radius **k** centered at the origin $O(0,0)$
- Plot the point $A(k,0)$
- Plot another point A' such that $\angle AOA' = \mathbf{a}$ radian
- Rename the point A' as P
- Show the coordinates of P
- Join OP using a line segment

  What does the coordinates of the point P represent ?

  Find the domain and range of $k\sin(x)$ and $k\cos(x)$ for different values of **k**
- Open Graphics 2 and plot the graphs of $k\sin(x)$ and $k\cos(x)$ as we done in Activity 4.2.
- Save this file as Activity 4.A

Activity 4.B $k\sin(2x)$

Discussion :

We construct an applet using which we describe the functions $k\sin(2x), k\cos(2x)$ etc.

Procedure :

- Open Activity 4.A and save it as Activity 4.B
- Edit the rotation of P as **2a** (Double click and edit as `Rotate(A,2a,O)`)

  What does the coordinates of the point P represent ?
- Open Graphics 2 and plot the graphs of $k\sin(2x)$ and $k\cos(2x)$
- Create an applet to describe $k\sin(ax)$ and $k\cos(ax)$, for different values of **k** and **a**

Lab 5

Trigonometric Identities

Aim

- To construct applets to establish the relation among various trigonometric functions like $\sin\left(\frac{n\pi}{2} \pm x\right)$ with $\sin x$, $\cos x$ etc.
- To confirm the findings geometrically

Concepts

- Trigonometric functions are defined by means of coordinates of a point on the unit circle centred at origin
- Concept of congruent triangles

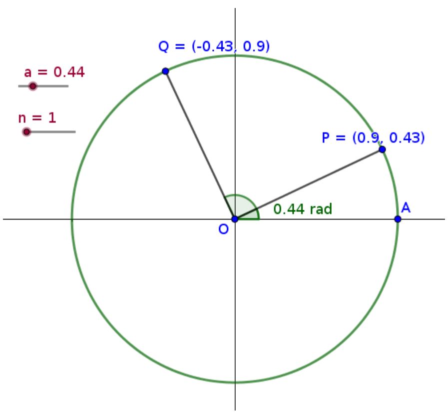
Discussion

Similar to the applet constructed in Lab 4 we construct two points on the unit circle centred at the origin. One point P has a rotation of x radians from $(1, 0)$ and the second point Q has a rotation of $\frac{n\pi}{2} + x$. Comparing the coordinates of P and Q we establish the relation between $\sin(\frac{n\pi}{2} + x)$, $\cos(\frac{n\pi}{2} + x)$, $\tan(\frac{n\pi}{2} + x)$ with $\sin x$, $\cos x$, $\tan x$ etc. for different integral values of **n**. We repeat the activity by changing the rotation of Q as $\frac{n\pi}{2} - x$.

Activity 5.1 $\sin(\frac{n\pi}{2} + x)$

Procedure:

- As in Activity 4.1, draw a unit circle centred at the origin. (With the same initial settings as in Activity 4.1)
- Take a point $A(1, 0)$. Create a slider **a** with min $= 0$ and plot the point P on the circle such that $\angle\, AOP = a$
- Create an integer slider **n** with minimum 1 and maximum 8
- Plot another point Q on the circle such that $\angle\, AOQ = \frac{n\pi}{2} + a$ (Using `Angle with Given Size` tool; click on A and O in that order and give `n*pi/2+a` in the box provided for entering the angle. We get a new point A', rename it as Q)



For different values of **a**, observe the coordinates of P and Q and complete the following table.

Sl.No	a	$\sin a$	$\cos a$	$\sin(\frac{\pi}{2}+a)$	$\cos(\frac{\pi}{2}+a)$	$\sin(\pi+a)$	$\cos(\pi+a)$	$\sin(\frac{3\pi}{2}+a)$	$\cos(\frac{3\pi}{2}+a)$

Establish the relation between $\sin(\frac{n\pi}{2}+x)$, $\cos(\frac{n\pi}{2}+x)$, $\sin x$ and $\cos x$ for different values of n

Do you observe any peculiarity for even and odd values of n ?

What is the relation between $\tan(\frac{n\pi}{2}+x)$ and $\tan x$?

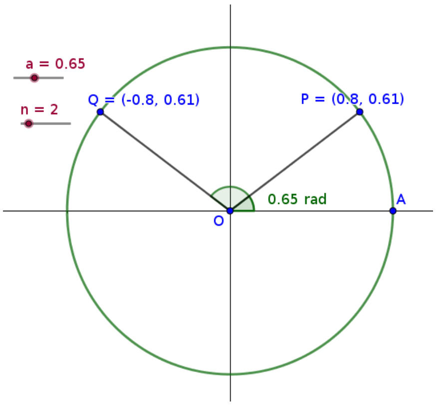
- Save this file as Activity 5.1

Activity 5.2 $\sin(\frac{n\pi}{2}-x)$

Procedure:

- As in Activity 5.1, draw a unit circle centred at the origin. Plot points A and P.
- While plotting Q, give angle as `n*pi/2-a`
- Show the coordinates of P and Q

  (Or you can edit the applet of Activity 5.1 as follows- Open the applet, using *save as* option from *file* menu, save the applet as Activity 5.2. Then double click on Q and change the angle as `n*pi/2-a`)



For different values of a, observe the coordinates of P and Q and complete the following table.

Sl.No	a	$\sin a$	$\cos a$	$\sin(\frac{\pi}{2}-a)$	$\cos(\frac{\pi}{2}-a)$	$\sin(\pi-a)$	$\cos(\pi-a)$	$\sin(\frac{3\pi}{2}-a)$	$\cos(\frac{3\pi}{2}-a)$

Establish the relation between $\sin(\frac{n\pi}{2} - x)$, $\cos(\frac{n\pi}{2} - x)$, $\sin x$ and $\cos x$ for different values of n

Do you observe any peculiarity for even and odd values of n ?

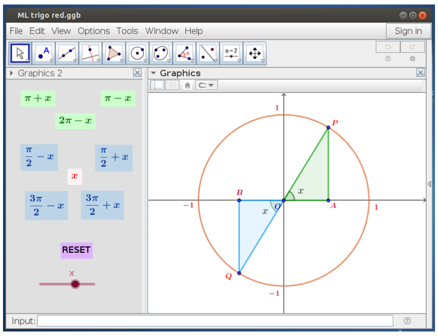
What is the relation between $\tan(\frac{n\pi}{2} - x)$ and $\tan x$

- Save this file as Activity 5.2

Activity 5.3 Geometrical Proof

Procedure:

Use Applet ML 5.3



About the applet:

Using the slider **x** we can rotate the point P by an angle x along the unit circle.
Using the buttons, we can select the rotation of the point Q as $\pi + x$, $\pi - x$, $\frac{\pi}{2} + x$ etc.

With the help of this applet, try to give a geometrical proof of the result that we found on activities Activity 5.1 and Activity 5.2 (Hint:Use properties of congruent trianges)

Additional Activities

Activity 5.A $\cos(x + y)$

Discussion :

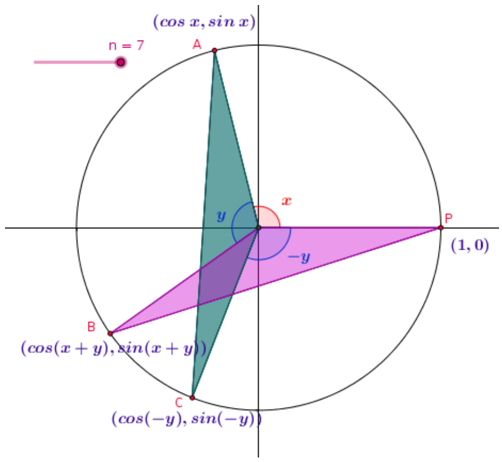
Using an applet we prove the result
$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

Procedure :

Use Applet ML 5.A

About the applet

By moving the slider **n** from 1 to 7 we get a step by step construction of the given figure.



Using congruence of triangles prove the result

$$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

Activity 5.B Simple Harmonic Motion

Discussion :

Simple Harmonic Motion (SHM) is a periodic function for which the displacement is a sinusoidal function of time (displacement can be expressed as a function of *sine* or *cosine*). The equation

$$x(t) = A\cos(\omega t + \phi)$$

represents an SHM with amplitude A , angular frequency ω and initial phase ϕ. We can identify an SHM as a projection of a uniforn circular motion on a straight line.

Procedure :

- Make the initial settings as in Activity 4.1
- Create sliders **A**, ω with Min = 0 and Max = 10, ϕ and t with Min = 0, Max = 50, Increment 0.01.
- Draw a circle of radius A centred at the origin and plot the point $O(0, 0)$.
- Plot the point $B(A, 0)$
- Plot the point B' such that $\angle\, BOB' = \phi$ radian
- Plot the point B'' such that $\angle\, B'OB'' = \omega\mathbf{t}$ radian. (Now the rotation of B'' from B is $\omega t + \phi$).
- Draw OB''
- Draw a perpendicular from B'' to x axis and plot the point of intersection C. Hide the perpendicular line and draw $B''C$ with a line segment.
- By animating the slider t, we can see that the point C moves in SHM along the x axis.

Write x coordinates C in terms of A, ω, ϕ and t.

What is the x coordinate of C when $t = 0$?

If x represents the displacement of the point C from the origin at time t, write its equation of motion.

Lab 6

Solutions of Trigonometric Equations

Aim

- To use graphs of trigonometric functions to find solutions of trigonometric equations.

Concepts

- Principal and general solutions of Trigonometric Equations

Discussion

The point at which the graph of the function $f(x)$ meets the x axis gives the solutions of the equation $f(x) = 0$.

The solutions of $f(x) = a$ are given by the x coordinates of the points of intersection of the curves $y = f(x)$ and $y = a$ or the x coordinates of the points of intersection of the curve $y = f(x) - a$ with the x axis. For the convenience of finding the x coordinate, we use the second method.

Activity 6.1 Solution of $\sin x = a$

Procedure:

- Initial settings - to change the distance marking on the x axis
  Create an integer slider **n**, with min=1. Right click anywhere on the graphics view
  `Graphics` → `x axis` → `distance` → Type pi/n in the box
- Create a slider **a** with increment 0.01
- Draw the graph of $f(x) = \sin x - \mathbf{a}$
- The points at which this graph cuts or touches the x axis gives the solution of the equation $\sin x = \mathbf{a}$

  Set $\mathbf{a} = 0$ and find the solutions of the equation $\sin x = 0$

  Gradually increase the value of **a**, and observe how the above points deviate from multiples of π

- Create an input box for f

Find the principal and general solutions of the equations given in the following table. If needed, you can change the distance on the x axis using the slider **n**. (Use input box of f to change the function).

Sl. No	Trig. Equation	Principal Solutions	General Solution
1	$\sin x = \frac{1}{2}$		
2	$\sin x = \frac{\sqrt{3}}{2}$		
3	$\sin x = \frac{-1}{2}$		
4	$\sin 2x = \frac{1}{2}$		
5	$\sin 3x = \frac{-\sqrt{3}}{2}$		
6	$\sin x = \frac{1}{\sqrt{2}}$		
7	$\sin(\frac{x}{3}) = \frac{\sqrt{3}}{2}$		

Activity 6.2 Solution of $\cos x = a$

Procedure:

- In the above applet, change the function to $\cos x - \mathbf{a}$

By observing the points at which the graph cuts or touches the x axis, find the principal and general solutions of the equations given in the following table.

Sl. No	Trig. Equation	Principal Solutions	General Solution
1	$\cos x = \frac{1}{2}$		
2	$\cos x = \frac{\sqrt{3}}{2}$		
3	$\cos x = \frac{-1}{2}$		
4	$\cos 2x = \frac{1}{2}$		
5	$\cos 3x = \frac{-\sqrt{3}}{2}$		
6	$\cos(\frac{x}{2}) = \frac{1}{\sqrt{2}}$		
7	$\cos(\frac{x}{3}) = \frac{1}{2}$		

Activity 6.3 Solution of $\tan x = a$

Procedure:

Using the above applet, find the principal and general solutions of the following equations.

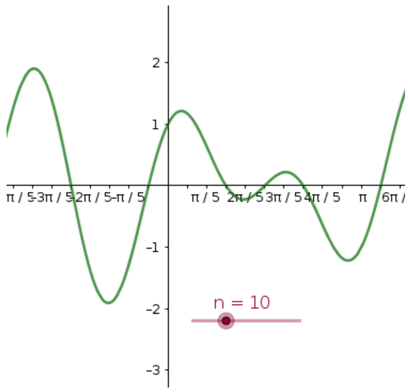
Sl. No	Trig. Equation	Principal Solutions	General Solution
1	$\tan x = 1$		
2	$\tan x = \sqrt{3}$		
3	$\tan 2x = \frac{1}{\sqrt{3}}$		
4	$\tan 3x = -1$		
5	$\tan x = -\sqrt{3}$		
6	$\tan(\frac{x}{2}) = -\sqrt{3}$		
7	$\tan(\frac{x}{4}) = \sqrt{3}$		

Save the applet as Activity 6.3

Activity 6.4 Solution of trigonometric equations in general

Procedure:

- Create an integer slider **n**
- Change the distance mark on the x axis as $\frac{\pi}{n}$
- Input $f(x) = \sin x$ and $g(x) = \cos x$
- Create input boxes for f and g.
- Using the input command `f-g`, draw the graph of $f(x) - g(x)$.
- Hide the graphs of f and g



Observe the points of intersection of the graph of $f(x) - g(x)$ with the x axis, which gives the solution of the equation $f(x) = g(x)$. Hence find the principal and general solutions of the following equations (To confirm your answer, you may change the distance on the x axis using slider **n**)

Sl. No	Trig. Equation	Principal Solutions	General Solution
1	$\sin x = \cos x$		
2	$\sin 2x = \cos x$		
3	$\cos 2x = \sin x$		
4	$\cos 2x = \cos x$		
5	$\sin 2x + \cos x = 0$		
6	$\sin 2x + \sin 3x = 0$		

Lab 7

Shifting and scaling of graphs of Trigonometric Functions

Aim

- To study the effect of the constants **a**, **b** and **c** of the trigonometric function $a\sin(bx+c)$ on the graph of the function.

- To study the periodicity of trigonometric functions.

Concepts

- Graphs of trigonometric functions

- Periodicity of trigonometric functions

Discussion

We have already seen the translation and reflection of the graph of a function according to some changes in the definition of the function. Here we discuss the scaling of the graph of trigonometric functions along with translation and reflection. These concepts will be useful in the study of waves in Physics.

Activity 7.1 Shifting

Procedure:

- Change the distance on the x axis in terms of π.

- Draw the graph of $f(x) = \sin x$.

- Create a number slider **a** with min = -10, max = 10 and increment 0.01

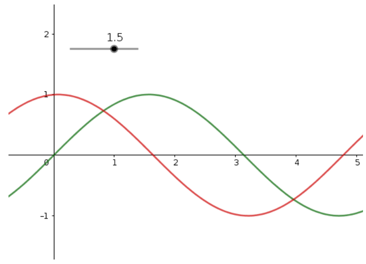
A function f for which there exist a real number a such that $f(x+a) = f(x)$, for all x is called a periodic function. The smallest positive number satisfying this identity is called the period of f.

- Draw the graph of $\sin(x+a)$ (By giving input $f(x+a)$)

Find the minimum positive value of **a**, for which $\sin(x+a) = \sin x$ (This value of **a** is called period of $\sin x$)

Observe the change in the graph according to **a**. (Refer Activity 2.2)

- Set the value of slider **a** at 0
- Draw the graph of $\cos x$
- Move the slider **a** and compare the graphs of $\sin(x + a)$ and $\cos x$.



Can you predict the value of **a** for which the graph of $\sin(x + a)$ coincides with that of $\cos x$?

- Draw the graphs of the following functions (Set-1)
  1) $\sin(\frac{\pi}{2} - x)$ 2) $\sin(\frac{\pi}{2} + x)$ 3) $\sin(\pi - x)$ 4) $\sin(\pi + x)$
  5) $\sin(\frac{3\pi}{2} - x)$ 6) $\sin(\frac{3\pi}{2} + x)$ 7) $\sin(2\pi - x)$ 8) $\sin(2\pi + x)$
- Also draw the graphs of $\sin x$, $-\sin x$, $\cos x$, and $-\cos x$ (Set-2)

Compare the graphs of functions in Set-1 with the graphs of functions in Set-2 and note down the observations in the following table

Sl. No	Trig. Function	Reduced form
1	$\sin(\frac{\pi}{2} - x)$	
2	$\sin(\frac{\pi}{2} + x)$	
3	$\sin(\pi - x)$	
4	$\sin(\pi + x)$	
5	$\sin(\frac{3\pi}{2} - x)$	
6	$\sin(\frac{3\pi}{2} + x)$	
7	$\sin(2\pi - x)$	
8	$\sin(2\pi + x)$	

You can hide or show graphs (or any object) by clicking on the bullets in the Algebra view

- Draw the graphs of the following functions (Set-3)
  1) $\cos(\frac{\pi}{2} - x)$ 2) $\cos(\frac{\pi}{2} + x)$ 3) $\cos(\pi - x)$ 4) $\cos(\pi + x)$
  5) $\cos(\frac{3\pi}{2} - x)$ 6) $\cos(\frac{3\pi}{2} + x)$ 7) $\cos(2\pi - x)$ 8) $\cos(2\pi + x)$

Compare the graphs of functions in Set-3 with the graphs of functions in Set-2 and note down the observations in the following table

Sl. No	Trig. Function	Reduced form
1	$\cos(\frac{\pi}{2} - x)$	
2	$\cos(\frac{\pi}{2} + x)$	
3	$\cos(\pi - x)$	
4	$\cos(\pi + x)$	
5	$\cos(\frac{3\pi}{2} - x)$	
6	$\cos(\frac{3\pi}{2} + x)$	
7	$\cos(2\pi - x)$	
8	$\cos(2\pi + x)$	

Activity 7.2 Scaling

Procedure:

- Draw the graph of $f(x) = \sin x$
- Create a slider **a**
- Draw the graph of $a \sin x$ (By giving input $a * f$)

  Observe the change in the graph according to **a**

  How does the value of **a** affect the domain and range of the function af

- Change the function f to $\cos x$

  Repeat the above observations and make notes

Activity 7.3 Periods of Trigonometric Functions

Procedure:

- Draw the graph of $f(x) = \sin x$
- Create an integer slider **n**
- Draw the graph of $\sin(nx)$ (By giving input $f(nx)$)

  Observe the change in the graph according to **n**

  Write the periods of the following functions

  1) $\sin 2x$ 2) $\sin 3x$ 3) $\sin 5x$

  If **n** is a positive integer, what is the period of $\sin nx$

- Draw the graph of $\sin(\frac{x}{n})$ (Hide all other graphs and input `f(x/n)`)

  Identify the periods of $\sin(\frac{x}{2})$, $\sin(\frac{x}{3})$, $\sin(\frac{x}{4})$ etc.

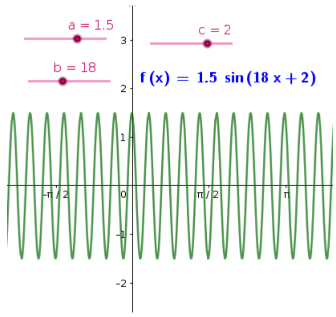
- Change the function f to $\cos x$

  Repeat the above observations and make notes

From the graph of $\sin x$, we can see that its period is 2π. Observe the graph of $\sin x$ in the intervals $[0, 2\pi]$, $[2\pi, 4\pi]$, $[4\pi, 6\pi]$ etc. We realise that the portions of the graph are identical in these intervals. Note that these intervals are of length 2π. We can also observe that the portions of the graph are not identical in any adjacent intervals with length less than 2π

Activity 7.4 Shifting and Scaling

Procedure:



- Draw the graph of $f(x) = \sin x$
- Create three sliders **a**,**b** and **c**
- Draw the graph of $a\sin(bx + c)$ (By giving input `a*f(b*x+c)`)

  Observe the changes in the graph according to the changes in the values of the sliders

- Change the function f to $\cos x$

  Repeat the above observations and make notes

Additional Activities

Activity 7.A Waves

Discussion :

This activity is related to the topic 'Waves' in Physics. A wave, when viewed mathematically is a function of the displacement x from the origin and time t and is expressed as

$$f(x, t) = a\sin(kx \pm \omega t)$$

where a is the amplitude of the wave, ω is the frequency of the wave and k is a scaling factor. In this activity we explore the geometrical nature of a wave by making use of the wave equation.

Procedure:

- Create four sliders **a**,**k**,ω,**t** all with min=0 and max=10, 500, 10 and 100 respectively.
- Draw the graph of $f(x) = a\sin(kx - \omega t)$
- Animate **t**, to get a propagating wave with amplitude **a**
- Draw the graph of $g(x) = a\sin(kx + \omega t)$
- Compare the waves f and g. What do you observe?
- Input $f + g$ which gives the resultant of the above waves, which is a standing wave. (You may learn more about waves in Physics class.)
- Change the values of **k**, ω and **t** and observe the changes and make notes.

Activity 7.B Music and Maths

Discussion :

Being a wave, sound can be represented in terms of trigonometric functions. Using GeoGebra we can produce sound of required frequency and amplitude.

Procedure:

- Create a slider **a** with minimum value 0 and maximum value 5
- Create an integer slider **n** with minimum value 0 and maximum value 1500
- Draw the graph of $f(x) = a\sin(n.2\pi x)$. We get a sine wave of amplitude **a** and frequency **n**. (You may learn more about sound in Physics class.)
- You can play the corresponding sound using `PlaySound` command. For this, create a button with caption Play (Take the Button tool and click on the Graphics View)
  Write the command as `PlaySound[f,0,100]` in the Scripting tab.
  (Which means, on clicking the button, it will play the pure sine tone of frequency **n** corresponding to the function f between 0 to 100).
  To stop the sound, take another button with caption Stop and write the script as `PlaySound[False]`.
  Clicking on the button will stop playing the sound
- Change the value of **a**, and observe the difference in the sound
- Change the value of **n** and observe the change in frequency of the sound. (You can verify the frequency of sound using a pitch analyser which is available in smart phones as mobile app.)

Activity 7.C Harmonic Sounds

Discussion :

In this activity we explore the superposition property of waves. This is achieved by adding two or more functions representing waves. We also discuss beats, which is observed when two sound waves of close frequency are superimposed. Similarly we discuss harmonic sounds, sounds whose frequency is an integer multiple of a fundamental frequency.

Procedure:

- Create two sliders **a** and **b** with minimum value 0 and maximum value 5
- Create two integer sliders **m** and **n** with minimum value 1 and maximum value 1500
- Input two functions $f(x) = a\sin(n.2\pi x)$ and $g(x) = b\sin(m.2\pi x)$
- Create the function $h = f + g$
- Create 4 buttons with captions Sound 1, Sound 2, Resultant and Stop with scripts as follows
  Sound 1 $\rightarrow$ `PlaySound[f,0,100]`
  Sound 2 $\rightarrow$ `PlaySound[g,0,100]`
  Resultant $\rightarrow$ `PlaySound[h,0,100]`
  Stop $\rightarrow$ `PlaySound[False]`

Set $\mathbf{m} = \mathbf{n} = 250$. Then Sound 1, Sound 2 and Resultant will play sounds of the same frequency

Set $\mathbf{m} = 250$, $\mathbf{n} = 251$. While playing Resultant, we experience a beat sound of frequency 1 (We can experience the beat sound of frequency 1 with any two frequencies of numerical difference 1)

Set $\mathbf{m} = 250$, $\mathbf{n} = 252$. While playing Resultant, we experience a beat sound of frequency 2

Set $\mathbf{m} = 250$, $\mathbf{n} = 500$ (or any integer multiple of 250) and play the resultant sound. Pitch analyser will show the frequency as 250. These two sounds are said to be harmonic.

You can repeat this exercise for any number of sounds.

Activity 7.D Blood Pressure

Each time your heart beats, your blood pressure first increases and then reduces as the heart rests between beats. The maximum and minimum blood pressures are called systolic and diastolic pressures respectively. Your blood pressure reading is written as systolic/diastolic. A reading of 120/80 is considered as normal.
A certain person's blood pressure was written as

$$p(t) = 115 + 25\sin(160\pi t)$$

where $p(t)$ is the pressure in mm of mercury at time t measured in minutes.

- Draw the graph of this function

Find the period of this function

How many times does his heart beat per minute?

Is this person's blood pressure normal?

Lab 8

Straight lines

Aim

- To establish the role of coefficients and constant in the general equation of a straight line
- To explore geometrically the Normal form of a straight line
- To explore geometrically a family of straight lines

Concepts

- General equation of a straight line
- Family of straight lines
- Normal form of a straight line

Discussion

A straight line is represented as $ax + by + c = 0$ where a,b and c are constants. If c is changed, while keeping a and b fixed, the straight line will change in a particular manner. Similar situations are encountered with other constants. In this activity we explore the variations in the coefficients and constant and its effect in the geometry of straight line.

All the above mentioned activities will result in a set of straight lines having a common property. This set is called a family of straight lines.

We also discuss the family of straight lines passing through a point of intersection of given lines.

Activity 8.1 General Form of Straight Lines

Procedure:

- Create three sliders **a**,**b** and **c**
- Draw the line $ax + by + c = 0$
- Change the values of **a**,**b** and **c**

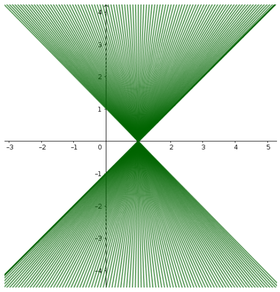
What happens to the line if

(i) $\mathbf{a} = 0$ (ii) $\mathbf{b} = 0$ (iii) $\mathbf{c} = 0$

(iv) $\mathbf{a} = \mathbf{b}$ (v) $\mathbf{a} = \mathbf{-b}$

Make the following changes and observe the corresponding changes in the line (Trace option of the line may be used).

1. Change **a** alone
2. Change **b** alone
3. Change **c** alone



Activity 8.2 Intersection of Two Lines

Procedure:

- Draw two lines say, $3x - 2y + 4 = 0$ and $2x + 5y - 6 = 0$
- Create a slider **k**
- Input the equation $3x - 2y + 4 + \mathbf{k}(2x + 5y - 6) = 0$

  It represents a line (why?)

  Change the value of **k**. Observe how the third line changes

  For what value of **k**, the third line coincides with the first line?

  For what value of **k**, the third line coincides with the second line?

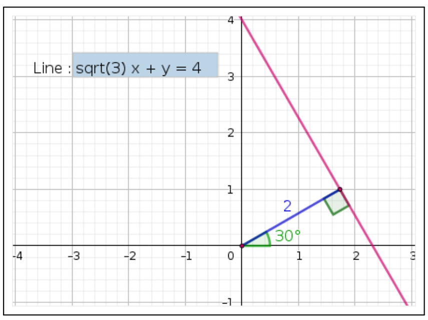
- Edit the equations of the first two lines so that they are parallel

  What happens to the third line for different values of **k**

Activity 8.3 Normal Form

Procedure:

- To find the normal form of a line geometrically, draw the line and the perpendicular from the origin to the line (use `Perpendicular Line` tool).
- Mark the point of intersection of the perpendicular with the line.
- Hide the perpendicular line and draw a line segment from the origin to the line.



- Show the length of the perpendicular and the angle made by it with the positive direction of the x axis.
- Create an input box for the line
- Save the applet as Activity 8.3

Using **Angle** tool we can show the angle in the following ways

- Click on the positive direction of x axis and then on the line segment. (Make sure that the line segment has been drawn starting from the origin, otherwise we won't get the required angle).
- Take a point - say C - on the positive direction of the x axis. Using the tool click on C, the origin and the point of intersection in that order. Hide C

Using this applet, write the normal form of the following lines

1. $x - \sqrt{3}y - 8 = 0$
2. $x - y - 2 = 0$
3. $\sqrt{3}x - y + 8 = 0$
4. $2x - 3y + 4 = 0$
5. $x + y = 5$
6. $5x + 2y + 3 = 0$

Write the equation of the lines in normal form for different values of ω and p (p is the distance of the line from the origin and ω is the angle made by the normal with the positive direction of x axis). Verify your answer using the above applet

Sl.No	Value of ω	Value of **p**	Equation of the line
1	$0°$	3	
2	$30°$	4	$\frac{\sqrt{3}x}{2} + \frac{y}{2} = 4$
3	$30°$	5	
4	$60°$	2	
5	$90°$	4	
6	$120°$	4	
7	$150°$	4	

Activity 8.4 Shifting of Origin

Procedure:

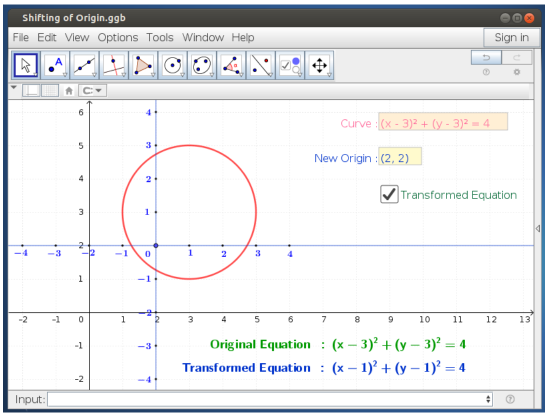
- Use the Applet ML 8.4

  About the applet:

  – You can click and drag at the origin to shift the axes.

  – You can change the curve and the origin using corresponding input boxes

  – You can see the transformed equation using check box



Shift the origin, parallel to the x axis or y axis and observe the changes in the new equation of the circle

What should be the new origin to get the transformed equation as $x^2 + y^2 = 4$. Guess the answer and check it.

Find the transformed equation, if the origin is shifted to the point $(1, 3)$. Check the answer.

Lab 9

Conic Sections

Aim

- To show conics as the section of a cone as well as the locus of a point

Concepts

- Cone and its section by a plane
- Locus of a point

Discussion

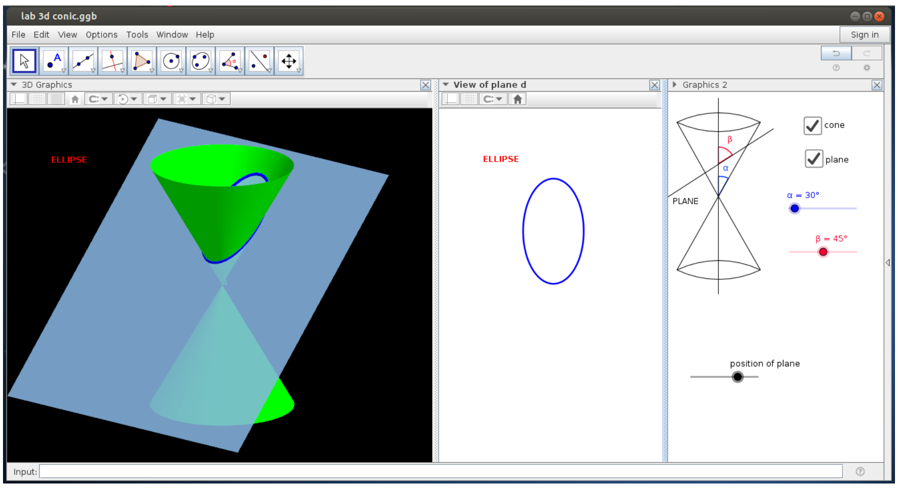
Conic sections are curves obtained by the intersection of a double cone by a plane. The angle at which the plane cuts the cone determines the curve. The semi vertical angle of the cone, the position at which the plane cuts the cone etc.. will determine the shape of the curve.

We treat the curves as the locus of a point moving on a plane subjected to certain constraints.

Activity 9.1 Cutting of a Cone by a Plane

Use the Applet ML 9.1

About the applet:



In this applet we can see 3 open windows. **Graphics 2** , **3D Graphics** and the third one is **View of plane d**

Graphics 2

- There are three sliders and two Check Boxes here.
- Using slider α we can change the semi vertical angle of the cone.
- Using slider β we can tilt the plane
- Using slider 'Position of the plane' we can change the position of the plane.
- Using the Check Boxes we can show or hide the cone and the plane.

3D graphics

We can see the 3D view here, using `Rotate 3D graphics View` tool, we can rotate the entire view to see it from a convenient angle.

View of plane d

We can see the curve, obtained by intersecting the cone with the plane.

Procedure:

Change the value of β for a fixed α. Observe the curves for different values of β.

α	**Curve**	β
25°	Circle	90°
	Parabola	25°
	Ellipse	$25° < \beta < 90°$
	Hyperbola	
30°	Circle	
	Parabola	
	Ellipse	
	Hyperbola	
45°	Circle	
	Parabola	
	Ellipse	
	Hyperbola	
50°	Circle	
	Parabola	
	Ellipse	
	Hyperbola	

For what values of β do we get the curves - circle, ellipse, parabola and hyperbola?

Change the position of the plane and observe the corresponding change in the shape of the curves

Change α and observe corresponding change in the curves.

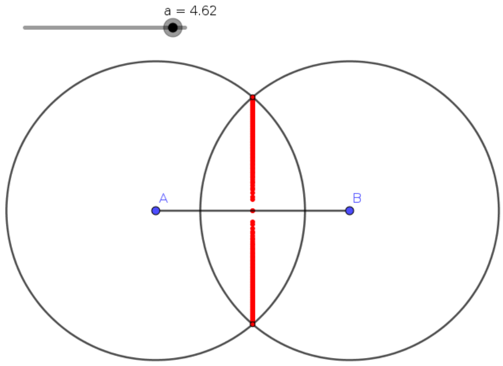
Activity 9.2 Locus of a point moving equidistant from two given points

Procedure:

- Plot two points A and B and join them (using `Segment` tool)
- Create a number slider **a** with min=0 and max=10
- Draw circles of radius **a** centred at A and B
- Plot the points of intersection of the circles and give trace of these points. (If necessary we can change the values of the slider so that the circles intersect each other)
- Give animation to the slider **a**

  Observe the path of the moving point. Describe the path.

- Save this file as Activity 9.2



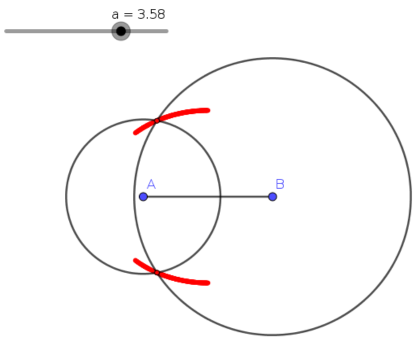
Adjust the increment of the slider as 0.01 or less to get a continuous path. If we use Shift key together with the arrow keys to move the slider, it will decrease the speed of the slider by one tenth. If we use Ctrl key, speed will be increased 10 times

Activity 9.3 Locus of a point the sum of whose distances from two given points is a constant

Procedure:

- As in Activity 9.2, create a slider **a**
- Plot two points A and B and join them.
- Draw a circle of radius **a** centred at A and another circle of radius $10 - \mathbf{a}$ centred at B
- Plot the points of intersection of the circles and give trace of these points.
- Give animation to the slider **a**
- Observe the path of the moving points. Identify the curve traced.
  (We can show the path of the moving point using `Locus` tool. For this, take the `Locus` tool, click on one of the points of intersection and on the slider. Similarly click on the other point of intersection and on the slider)

We can create this applet by editing the radius of the circle centred at B of the previous applet Activity 9.2



Change the distance between A and B and observe the change in the shape of the curve

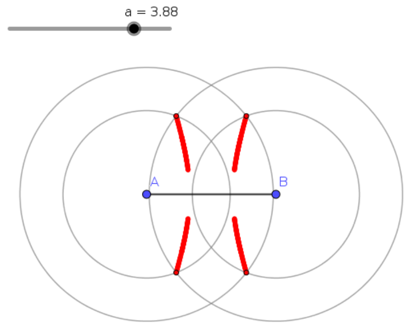
What happens to the curve when B approaches nearer and nearer to A?

What is the maximum possible distance between A and B to get a path?

Activity 9.4 Locus of a point the difference of whose distances from two given points is a constant

Procedure:

- As in Activity 9.2, Create a slider **a**
- Plot two points A and B
- Draw a circle of radius **a** centred at A and another circle of radius $\mathbf{a} + 4$ centred at B
- Plot the points of intersection of the circles and give their trace.
- Give animation to the slider **a**
- Draw another set of circles of radius **a** centred at B and of radius $\mathbf{a} + 4$ centred at A



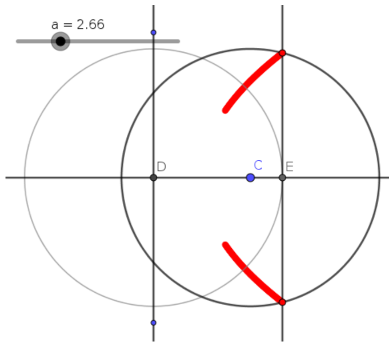
- Plot the points of intersection of these circles and give their trace.
- Give animation to the slider **a**

  Observe the path of the moving points. Identify the curve traced. (Here also we can use the Locus tool as in Activity 9.3)

Activity 9.5 Locus of a point equidistant from a fixed point and a fixed line

Procedure:

- Draw a line and plot a point C outside the line.
- Draw perpendicular to the line through C.
- Plot the point of intersection D of the line with its perpendicular.
- Create a slider **a** with min = 0, max = 15 and increment 0.01
- Draw a circle of radius **a** centred at D and plot its point of intersection E with the perpendicular line.
- Draw a line through E and parallel to the first line.



- Draw a circle of radius **a** centred at C and plot its points of intersection with the last line drawn. Trace on these points.
- Give animation to the slider **a**

  Observe the path of the moving points. Identify the curve traced.

- Draw the path using `Locus` tool

  Change the distance between the point and the first line and observe the change in the shape of the curve

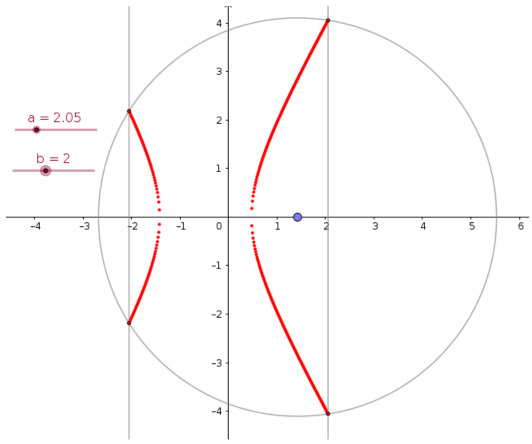
Additional Activities

Activity 9.A Focus - Directrix Definition

Discussion :

We discuss Parabola, Ellipse and Hyperbola as the locus of a point moving on a plane, keeping a specific ratio of distance from a fixed line and a fixed point.

Procedure :



- Create two sliders **a**, with min = 0 , max = 10 and increment 0.01 and **b** with min = 0 , max = 5 and increment 0.01
- Draw the line $x = 0$ and plot a point A on the positive side of the x axis.
- Draw the lines $x = a$ and $x = -a$
- Draw a circle of radius ab centred at A.
- Plot the point of intersection of the circle with the lines $x = a$ and $x = -a$ (If necessary, you can change the values of the sliders, so that the circle meets the lines).
- Trace the points of intersection
- Hide the axes, Set **b**=1 and animate slider **a**

  Observe the path of the points. Can you identify the curve ?

- Using `Locus` tool, draw the path of the points.

  Change the value of **b** and observe the path. Can you identify the curves for different values of **b** ?

  Try to define parabola, ellipse and hyperbola in terms of distances from a fixed line and a fixed point.

Activity 9.B Apollonius Circles

Discussion :

We discuss the locus of a point moving on a plane, keeping a specific ratio of distance from two fixed points.

Procedure :

- Create two sliders **a** and **r** with Min = 0 and increment 0.01. Create an input box for **a**.
- Plot two points A and B and join them
- Draw a circle of radius **r** centered at A and another circle of radius **ar** centered at B

- Plot the point of intersections of the circles and trace them. Find the locus of the points using `Locus` tool.

Observe the path for different values of **a**

Can you connect this with internal and external division of a line ?

Lab 10

Circle and Parabola

Aim

- To explore different methods of drawing Circles and Parabolas using GeoGebra tools and commands.

Concepts

- Definitions of Circle and Parabola
- Equations of Circle and Parabola

Discussion

Different tools and commands are available with GeoGebra for drawing Conic Sections according to the given data. To draw a given curve using a specific tool or command, the curve may be interpreted in a different form other than the given one. This needs a thorough knowledge about the curve.

We visualise some problems in Circles and Parabolas. We also discuss the change in the curve according to the constant in the standard equation of the Parabola.

Activity 10.1 Circle

We can draw a circle in different ways

- Centre and a point on the circle are given
  - Using `Circle with Centre through Point` tool, click on the centre and then on the point
  - Give input in the following manner. `Circle(centre point,point)`
- Centre and radius are given
  - Using `Circle with Centre and Radius` tool, click on the centre and enter radius.
  - Give input in the following manner. `Circle(Point,Radius)`
- Three points on the circle are given
  - Using `Circle through 3 Points` tool, click on the points
  - Give input in the following manner. `Circle(Point,Point,Point)`
- Input the equation of the circle
  For example: `(x-1)^2 + (y-2)^2 = 4` gives the circle $(x-1)^2+(y-2)^2=4$

Find the centre and radius of the following circles. Draw the circle and verify your answer. You can do it in any of the following ways.

- Draw the circle by direct input of the equation, find its centre and radius. Compare with your answer.

- Draw the circle using `Circle with Centre and Radius` tool, using the center and radius that you found. Compare its equation with the given equation.

> We can find the centre of a circle **c** using the input command **Center(c)** and its radius by the input command **Radius(c)**

1. $(x + 5)^2 + (y - 3)^2 = 36$
2. $x^2 + y^2 - 4x - 8y - 45 = 0$
3. $2x^2 + 2y^2 - 8 = 0$

Find the equations of the following circles. Input the equations obtained, draw the circles and verify your answer

1. Centre (-2,3) and radius 4
2. Centre (2,2) and passing through the point (4,5)

Construct the following circles without using `Circle through 3 Points` tool or input commands

1. Passing through the points (2,3) and (-1,1) and with centre on the line $x - 3y - 11 = 0$
2. Passing through the points (1,2) (5,4) and (3,6).
3. If three points are given, how can we find the equation of the circle passing through them (without using GeoGebra. Hint: above problem)

Activity 10.2 Parabola 1

Procedure:

- Using the tool `Parabola` select a line and a point to get a parabola with the line as directrix and the point as focus
- We can also draw a parabola using input command, for example, the input command `Parabola[(2,0),x+2=0]` gives the parabola with focus $(2, 0)$ and directrix $x + 2 = 0$
- If A represents a point and f represents a line then the command `Parabola[A,f]` gives the parabola with focus A and directrix f

Draw a line and plot a point. Draw the corresponding parabola. Change the distance between the line and the point, observe the corresponding change in the shape of the parabola

Find the focus and directrix of the following parabolas. Using `Parabola` tool, draw them. Check whether the equation of the parabola that you have drawn is same as the given equation.

i) $y^2 = 8x$ ii) $x^2 = 4y$

iii) $x^2 = -4y$ iv) $y^2 = -10x$

Activity 10.3 Parabola 2

Procedure:

- By giving the equation of the parabola directly in the input bar, we can draw the parabola
- Create a slider **a** and give the input $y^2 = 4ax$ and $x^2 = 4ay$

Change the value of **a** and observe the shape of the parabolas

Find the focus and length of latus rectum of the following parabolas. Verify your answer geometrically as follows;

Input the equation and draw the parabola. Using focus command (`Focus`[name of parabola]), we can find its focus. Draw the line through the focus and perpendicular to the axis of the parabola. Mark the points of intersection of this line with the parabola and join them with a line segment. Hide the line and measure the length of the latus rectum.

i) $y^2 = 6x$ ii) $x^2 = -8y$

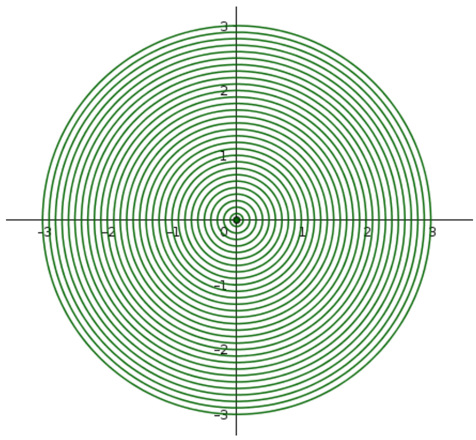
iii) $x^2 = 10y$ iv) $y^2 = -4x$

Additional Activities

Activity 10.A Family of Circles

Procedure:

- Input the following sequence command and draw the pattern
`Sequence[x^2+y^2=r^2,r,0,3,.1]` which gives a family of circles with centre at the origin and radius varying from 0 to 3 by an increment 0.1

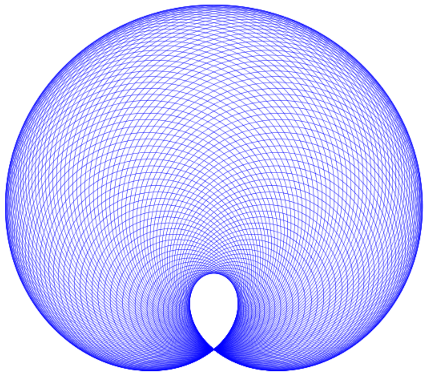


Imagine the pattern obtained by the following commands and then draw them

1. `Sequence[(x-r)^2+y^2=r^2,r,0,3,.1]`
2. `Sequence[x^2+(y+r)^2=r^2,r,0,3,.1]`

Draw the following family of circles using sequence command

1. Centres lie on the line $y = x$, and pass through the origin
2. Family of 100 circles of radius 3, whose centres lie at equal distance on the circle of radius 3 centred at the origin.(Hint: Use the concept of trigonometry-coordinates of points on a circle)
3. Family of 100 circles, whose centers lie at equidistant points on the circle of radius 3 centred at the origin and passing through the point $(3, 0)$
4. Do the above activity with a slider n to change the number of circles and another slider a so that all the circles pass through $(a, 0)$ or $(0, a)$ instead of $(3, 0)$. Change the value of a and observe the change in the pattern.



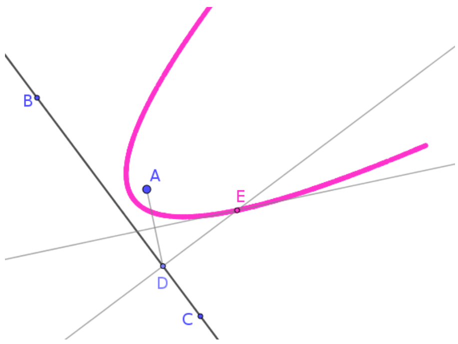
Activity 10.B Parabola with Given Focus and Directrix

Aim:

To create a parabola whose focus and directrix are given.

Procedure:

- Plot a point A and draw a line BC using `Line` tool.
- Take a point D on the line using `Point on Object` tool
- Draw the line segment AD and its perpendicular bisector
- Draw the line perpendicular to the first line and passing through D
- Take the point of intersection E of the above line with the perpendicular bisector and trace on this point



Animate point D and observe the path of E

Lab 11

Ellipse and Hyperbola

Aim

- To explore different methods of drawing ellipse and hyperbola using GeoGebra tools and commands.

Concepts

- Definitions of ellipse and hyperbola
- Equations of ellipse and hyperbola

Discussion

As in Lab 10 we use different tools and commands to draw Ellipse and Hyperbola. We need a thorough knowledge about the curve and its equation for drawing them with a specific tool or command. Sometimes it may need some calculations also.

Activity 11.1 Ellipse 1

Procedure:

If the foci and a point on the ellipse are known, we can draw it in the following way.

- Using `Ellipse` tool, select the foci one by one and then a point on the ellipse (or give input `Ellipse[focus,focus,point]`)

Using `Ellipse` tool draw the following ellipses

1. Foci $(\pm 3, 0)$, passing through the point $(5, 2)$
2. Foci $(0, \pm 4)$ and length of major axis 10
3. Foci $(\pm 2, 0)$ and length of minor axis 5
4. $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$
5. Using `Ellipse` tool draw the following ellipses and find the length of the latus rectum geometrically
   (a) Foci $(\pm 4, 0)$, passing through the point $(5, 2)$
   (b) $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

Activity 11.2 Ellipse 2

Procedure:

We can draw an ellipse if we know its foci and length of semi major axis using `Ellipse` command. For example, the command `Ellipse[(3,0),(-3,0),5]` gives the ellipse with foci $(\pm 3, 0)$ and length of semi major axis 5

Using the above command draw the following ellipses

1. $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$
2. Foci $(0, \pm 5)$, passing through the point $(2, 6)$

Create a slider **a**. Draw the ellipse using the command, `Ellipse[(-a,0),(a,0),5]`. Change the value of **a** and observe the corresponding change in the shape of the curve.

Activity 11.3 Hyperbola 1

Procedure:

If the foci and a point on the hyperbola are known, we can draw it in the following way.

- Using `Hyperbola` tool, select the foci one by one and then a point on the hyperbola (or give input `Hyperbola[focus,focus,point]`) to get the hyperbola with first the two points as foci and passing through the third point

  Using `Hyperbola` tool draw the following hyperbolas

  1. Foci $(\pm 3, 0)$, passing through the point $(5, 2)$
  2. Foci $(0, \pm 4)$ and length of transverse axis 6
  3. Foci $(\pm 3, 0)$ and length of conjugate axis 5
  4. $\dfrac{x^2}{16} - \dfrac{y^2}{25} = 1$
  5. Using `Hyperbola` tool draw the following hyperbolas and find the length of the latus rectum geometrically
     (a) Foci $(\pm 4, 0)$, passing through the point $(5, 2)$
     (b) $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$

Activity 11.4 Hyperbola 2

Procedure:

- We can draw a hyperbola, if we know its foci and length of transverse axis using `Hyperbola` command. For example,the command `Hyperbola[(3,0),(-3,0),2]` gives the hyperbola with foci $(\pm 3, 0)$ and length of transverse axis 4

  Using the above command draw the following hyperbolas

  1. $\dfrac{x^2}{25} - \dfrac{y^2}{9} = 1$
  2. Foci $(0, \pm 5)$, passing through the point $(2, 6)$

Additional Activities

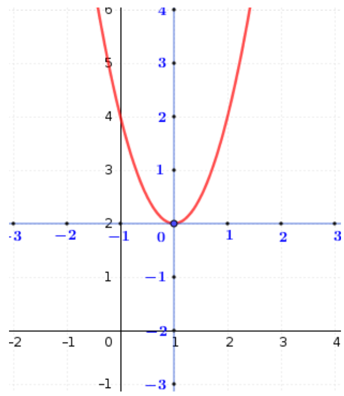
Activity 11.A Conic Sections in General

Aim:

To find the focus, directrix and length of the latus rectum of the parabola $y = 4x^2 - 2x + 5$

Procedure:

- Open Applet ML 8.4 and draw the parabola $y = 4x^2 - 2x + 5$.
- Shift the origin to the vertex of the parabola.



Find the transformed equation of the parabola

Using this, find its focus, directrix, and length of the latus rectum with respect to the new origin

Find the coordinates of the focus and the equation of the directrix with respect to the original system of axis

Find the equation of the ellipse with foci (-2,3) and (6,3) and passing through (5,5)

Find the equation of the hyperbola with foci (3,6) and (3,0) and passing through the origin

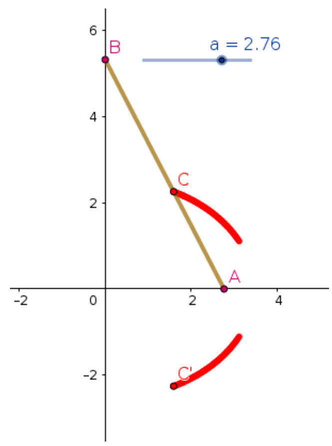
Activity 11.B Locus of a Point on a Sliding Rod

Aim:

To find the path of a point on a rod of fixed length (say, 6 units) sliding between two coordinate axes

Procedure:

- Create a slider **a** with min $= -6$, max $= 6$ and increment $= .01$
- Plot the point $A(a, 0)$
- Draw the circle with centre at A and radius 6
- Mark the point of intersection B of this circle with the y axis.
- Draw the line segment AB
- Hide the circle
- Plot a point C on the line segment and trace on it.



Animate slider **a**, and observe the path of C

- To get the complete curve, reflect C on the x axis and trace on it
- Using `Locus` tool, draw the locus of the point C and its reflection

What happens to the path if C is

- Near to A
- Near to B
- At the mid point of AB

Lab 12

Basics of 3D

Aim

- To explore the properties of points on the coordinate axes, coordinate planes and in different octants.
- To construct Three dimensional objects .
- To explore internal / external division of a line by coordinate planes.

Concepts

- Coordinate axes and coordinate planes
- Coordinates of a point in space
- Octants
- Internal / External division

Discussion

We discuss the properties of points on coordinate axes, coordinate planes and in different octants. We observe the movement of a point according to the change in its coordinates.

We construct rectangular boxes using the concept of 3D Geometry.

We discuss the method of finding by mere observation whether a coordinate plane divides a line joining two points internally or externally.

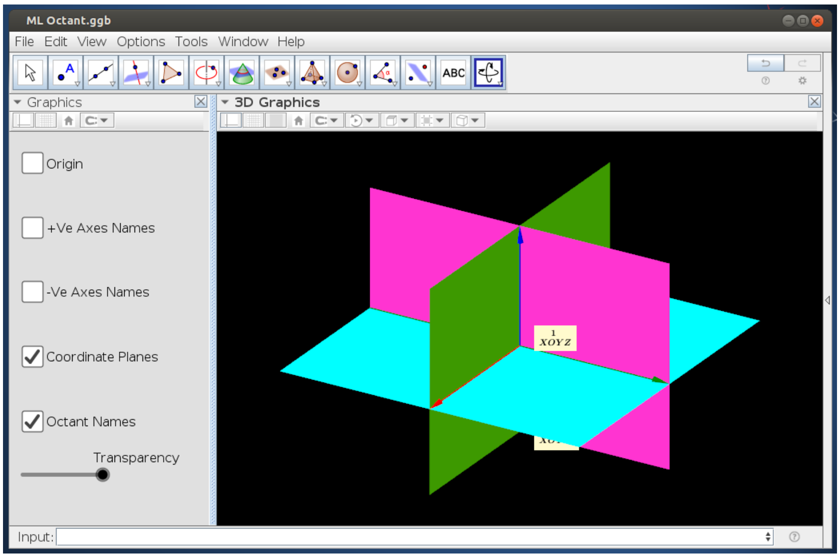
Activity 12.1 Octants

Procedure:

Use the Applet ML 12.1

About the Applet :

In this applet we can see the coordinate axes and coordinate planes.
Name and number of octants are given.
Using Rotate tool, we can rotate it in any direction.



Write the coordinates of some points lie on the x axis, y axis, z axis and plot them using input commands.

Write the coordinates of some points lie on the xy, yz, xz planes and plot them using input commands.

Write the coordinates of some points lie in the 1^{st}, 3^{rd}, 4^{th}, 7^{th}, 8^{th} octants and plot them using input commands.

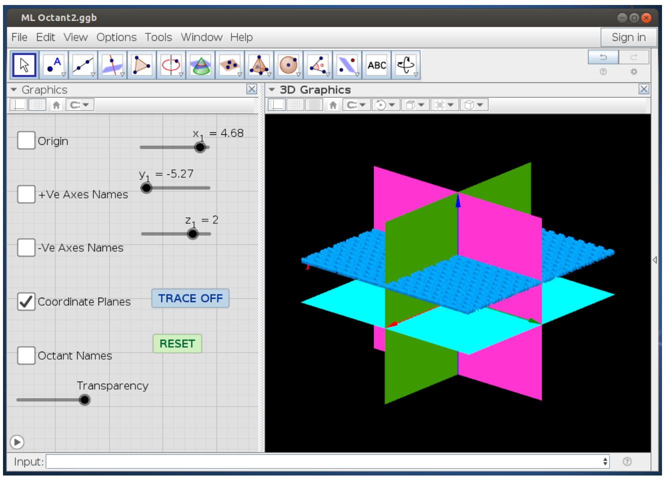
Activity 12.2 Movement of a Point

Procedure:

Use Applet ML 12.2

About the Applet :

This applet is similar to that we used in the previous activity. In addition to it, three sliders x_1, y_1 and z_1 are given in the Graphics View.
The point $P(x_1, y_1, z_1)$ is given in 3D Graphics. We can change its position using sliders.



Move the sliders according to the instructions given. Observe the movements of the points and complete the following table.

Sl. No	Movement and value of sliders	Movement of the point
1	$\mathbf{y}_1 = 0$, $\mathbf{z}_1 = 0$ and move $\mathbf{x}_1$	Moves along the x axis
2	$\mathbf{y}_1$ and $\mathbf{z}_1$ are any constants, move $\mathbf{x}_1$	
3	$\mathbf{x}_1$ and $\mathbf{z}_1$ are any constants, move $\mathbf{y}_1$	
4	$\mathbf{y}_1$ and $\mathbf{x}_1$ are any constants, move $\mathbf{z}_1$	
5	$\mathbf{z}_1 = 0$, move $\mathbf{x}_1$ and $\mathbf{y}_1$	
6	$\mathbf{z}_1 = 2$, move $\mathbf{x}_1$ and $\mathbf{y}_1$	
7	$\mathbf{y}_1 = 0$, move $\mathbf{x}_1$ and $\mathbf{z}_1$	
8	$\mathbf{y}_1$ any constant, move $\mathbf{x}_1$ and $\mathbf{z}_1$	
9	$\mathbf{z}_1$ any constant, move $\mathbf{x}_1$ and $\mathbf{y}_1$	

Activity 12.3 Box

Procedure:

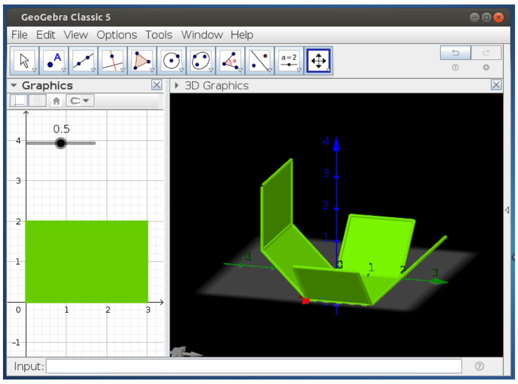
- Open 3D Graphics (`View` $\longrightarrow$ `3D Graphics`)
- Do the following initial settings
  `Options` $\longrightarrow$ `Labelling` $\longrightarrow$ `No New Objects`
- In the Graphics View, draw the rectangle (using `Polygon` tool) whose vertices are $(-1, 0)$, $(2, 0)$, $(2, 2)$ and $(-1, 2)$

> Using Net tool, we can open out the prism. For this choose
> *Net* $\longrightarrow$ *click on the prism*
> This generates a slider in the Graphics View. Moving this slider, we can fold and unfold the prism

- We can see this rectangle in 3D Graphics. Using `Extrude to Prism or Cylinder` tool, click on the rectangle from 3D Graphics, enter 4 as its altitude and click `OK`

  What are the coordinates of the vertices of the prism?

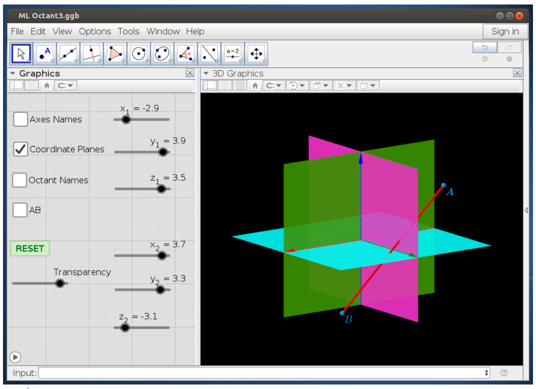
  Write the coordinates of the vertices of a prism, so that each vertex lies in different octants. Construct the prism.



Activity 12.4 Section of a Line by Coordinate Planes

Procedure:

Use Applet ML 12.4



About the Applet :

In this applet 6 sliders $\mathbf{x}_1, \mathbf{y}_1, \mathbf{z}_1$ and $\mathbf{x}_2, \mathbf{y}_2, \mathbf{z}_2$ are given.

The line segment AB joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is also given.

We can change the positions of A and B using sliders.

We can show/hide the extension of AB using the Check Box.

Adjust the values of sliders so that the points A and B do not coincide and both lie in the 1^{st} octant

Adjust any one slider so that the yz plane divides the line segment joining the points A and B internally

Adjust the sliders so that the xy and yz planes divide line segment AB internally

Adjust the sliders so that all the coordinate planes divide AB internally.

Additional Activities

Activity 12.A Construction of a Box by Cutting Squares from Corners and Folding up the Flaps

In Activity 3.B , we discussed about an open box made from a cardboard of size 5 m by 3 m, by cutting out squares of equal size from all the four corners of the cardboard and folding up the sides.

- Create such a box in which we can change the size of the square cut away, with the help of a slider.

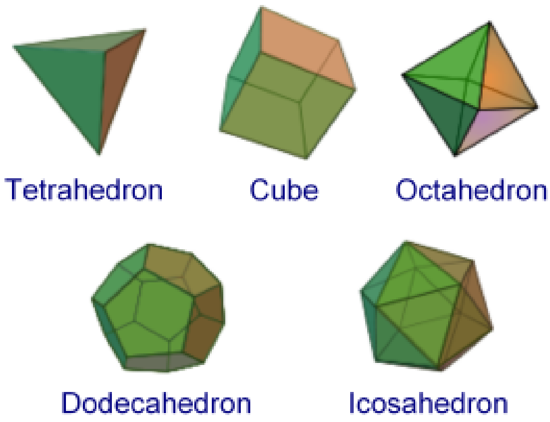
  Find the volume of the box using `Volume` tool

  Find the maximum volume of such a box and the length of the square that is to be cut away from the cardboard in order to get the maximum volume.

Activity 12.B Platonic Solids

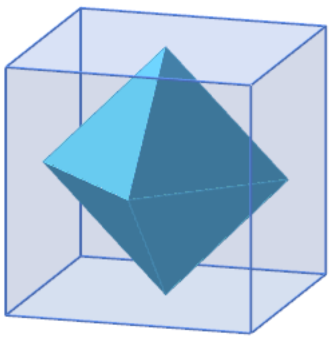
Discussion :

In three-dimensional space, a Platonic solid is a regular, convex polyhedron. It is constructed by congruent (identical in shape and size) regular (all angles equal and all sides equal) polygonal faces with same number of faces meeting at each vertex. Only five solids meet this criteria. Tetrahedron, Cube, Octahedron, Dodecahedron and Icosahedron.



Procedure :

- Cube is the most popular platonic solid. Construct a cube of side 3 units
- Construct a regular tetrahedron of side 3 units
- The polyhedron whose vertices are midpoints of faces of a cube is an Octahedron. Construct an octahedron of side 3 units. Learn more about Dodecahedron and Icosahedron and try to construct them.



Lab 13

Limits

Aim

- To explore geometrically the concept of the limit of a function at a point.

Concepts

- Value of a function at a point
- Graph of a function

Discussion

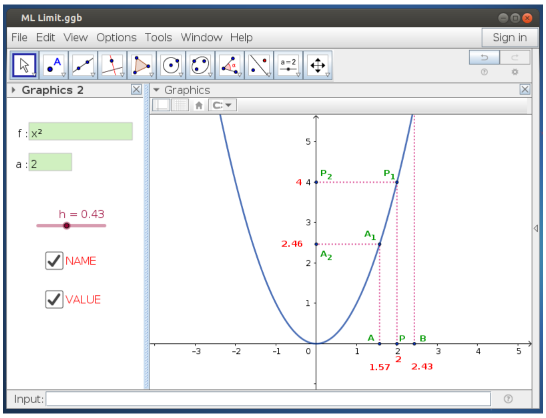
We explore geometrically the concept of limit at a point. We discuss the existence and different cases of non existence of limit, the nature of the graph at a point where limit exists/does not exist, the concept of left limit and right limit etc.

We interpret geometrically some standard limits also.

Activity 13.1 Geometrical Interpretation of Limits

Procedure:

Use the Applet ML 13.1



About the applet :

- You can see the graph of a function $f(x)$, 3 points A, B, P on the x axis, corresponding points A_1, B_1, P_1 on the graph and A_2, B_2, P_2 on the y axis
- ' NAME ' Check Box: By clicking on it you can show/hide the names of the points
- ' VALUE ' Check Box: By clicking on it you can show/hide the x coordinates of the points A, B and P and the y coordinates of the points A_2, B_2 and P_2
- Slider **h**: Using this we can bring the points A and B towards P
- Input Box **a**: To change the position of P
- Input Box **f**: To change the function

Initial settings

- $f(x) = x^2$
- $\mathbf{a} = 2$
- $\mathbf{h} = 1$
- Show the names of the points

Gradually change the value of **h** from 1 to 0. Observe the movements of the points. What happens to A_2 and B_2 as A and B approaches P ?

Show the values of the points. Set **h**=1 and gradually bring it to 0. Observe the values. What happens to the x coordinates of the points A and B? What happens to the y coordinates of A_2 and B_2?

We can record the value of points to spreadsheet as follows. Open spreadsheet view $\longrightarrow$ Spreadsheet. Right click on A_1 $\longrightarrow$ record to spreadsheet $\longrightarrow$ tick Row limit(10) $\longrightarrow$ Close. Similarly record the point B_1 to spreadsheet.

We can observe that as the x coordinates of A and B approach to 2, the y coordinates of A_2 and B_2 approach 4.
If we call the x coordinates of A and B as x, then the y coordinates of A_2 and B_2 are $f(x)$
So we observe that as $x \to 2$, $f(x) \to 4$
ie, the limit of $f(x)$ at $x = 2$ is 4

What happens to the points A, B, A_2 and B_2 when $\mathbf{h} = 0$?

Activity 13.2 Limit of Rational Functions

Procedure:

- In the above applet, change the function to $f(x) = \dfrac{x^2 - 4}{x - 2}$
- Move the slider **h** from 1 to 0

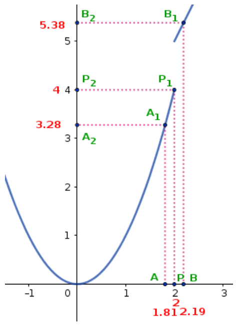
  What is the limit of this function at $x = 2$

  What happens to the points A_2 and B_2 when $\mathbf{h} = 0$ (Refer Activity 3.2)

Activity 13.3 Limit of Piecewise Functions

Procedure:

Using the above applet, discuss the limit of the following functions



1. $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 2x+1 & \text{if } x > 2 \end{cases}$ at $x = 2$

   (Input `If[x<=2,x^2,2x+1]` in the Input Box for the function f)

   What happens to $f(x)$ as x approaches to 2 from left and right?

2. Change $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 2x & \text{if } x > 2 \end{cases}$ and discuss the limit at $x = 2$

Discuss the existence of limit for the following functions

1. $f(x) = \dfrac{1}{x}$ at $x = 0$

2. $f(x) = \begin{cases} 1 & \text{if } x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$ at $x = 0$

3. $f(x) = \begin{cases} x-2 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ x+2 & \text{if } x > 0 \end{cases}$ at $x = 0$ (Input `If[x<0,x-2,x>0,x+2,0]` Or `If[x<0,x-2,x==0,0,x+2]`)

4. $f(x) = \begin{cases} x^2-1 & \text{if } x \leq 1 \\ -x^2-1 & \text{if } x > 1 \end{cases}$ at $x = 1$

Activity 13.4 Limit of Trigonometric Functions

Procedure:

Plot the graphs of $\sin x$ and x in the same graphics view. Zoom it at the origin. What do you see? What inference do you get from this?

Using the applet used in the previous activity, discuss the limit of $\dfrac{\sin x}{x}$ at $x = 0$

Plot the graphs of x^2, $\sin(x^2)$, $\sin^2 x$, $\tan(x^2)$ and $\tan^2 x$ on the same Graphics View. Zoom it at the origin. What do you see? What inference you get from this?

Activity 13.5 Limit of Exponential and Logarithmic Functions

Procedure:

- Input **a**=0. We get a slider in the Algebra view. Show it in the Graphics view by clicking on it
- Draw the graph of the function $f(x) = e^x - a$
- Input `y = x` to get the line
- Using `Reflect about Line` tool, click on the graph and on the line, we get the reflection of the graph of e^x on the line $y = x$, which represents the graph of $log_e(x)$

- Using the slider **a**, move the graph of f downwards until the line becomes tangent to the curve

  What happens to the reflection ?

  What are the definitions of the functions represented by the curves?

- Zoom it at the origin until the three curves seem to be one

  What do you infer from this?

  Write down some limits using this inference

Additional Activities

Activity 13.A Some more problems

Procedure:

With the help of the applet, discuss the limit of the following functions

1. $f(x) = \sin\left(\frac{1}{x}\right)$ at $x = 0$
2. $f(x) = x\sin\left(\frac{1}{x}\right)$ at $x = 0$
   Draw the lines $y = x$ and $y = -x$. Why does the graph of $x\sin\left(\frac{1}{x}\right)$ lie between these lines?
3. $f(x) = x^2\sin\left(\frac{1}{x}\right)$ at $x = 0$
   Draw the curves $y = x^2$ and $y = -x^2$. Discuss the existence of the limit of $x^2\sin\left(\frac{1}{x}\right)$ at 0 with the help of these graphs
4. $f(x) = \sqrt{x}\sin\left(\frac{1}{x}\right)$ at $x = 0$
   Draw the parabola $y^2 = x$. Discuss the existence of limit of the above function with the help of this curve

Lab 14

Derivative at a point

Aim

- To identify a tangent line in terms of secant lines.
- To explore the geometrical interpretation of the derivative of a function at a point.
- To explore different cases of non-differentiability of a function at a point.

Concepts

- Limit of a function
- Slope of a line passing through two points
- Secant line

Discussion

A secant line to a curve is a line joining two points on that curve. But a tangent line to a curve is not that much simple to define. In this lab we try to identify a tangent line in terms of secant lines. We discuss the geometrical interpretation of the derivative of a function at a point. We also discuss different cases of non-differentiability of a function at a point.

Activity 14.1 Geometrical Meaning of Derivative at a Point

Procedure:

Use the Applet ML 14.1

About the applet

In this applet, the graph of a function $f(x)$ is given ($f(x) = x^2$).

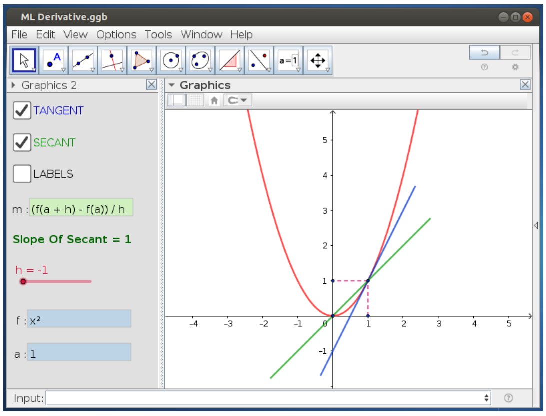
a is a number. We can change its value using the input box for it.

P is the point $(a, f(a))$, on the graph.

By clicking on the ' TANGENT ' checkbox, we can show/hide the tangent at P.

h is a slider and Q is the point $(a+h, f(a+h))$ on the graph.

By clicking on the ' SECANT ' checkbox we can show/hide the secant line (the line joining P and Q)



Set $f(x) = x^2$ and $\mathbf{a} = 1$.
Show the tangent line and the secant line

Gradually change the value of **h** from 1 to 0 and observe the secant line. What happens to it as **h** approaches 0 ?

Similarly change the value of **h** from -1 to 0 and observe the secant line. What happens to it as **h** approaches 0 ?

What happens to the secant line at $h = 0$?

Since the coordinates of P are $(a, f(a))$ and that of Q are $(a+h, f(a+h))$, the slope of the secant line is $m = \dfrac{f(a+h) - f(a)}{h}$.

We can find it using the input command `m = (f(a+h)-f(a))/h`

Observe the change in m as h approaches 0 from left and right. Find the number to which m approaches in each case. Hence find the slope of the tangent at P$(1, 1)$

Activity 14.2 Derivative at a Point

Procedure:

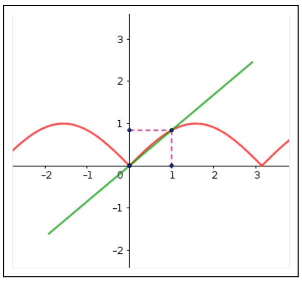
- Using the above applet,find the slope of the tangent to the curves given below at the given points, using the concept that the slope of the tangent is the limiting case of slope of a secant.

  1. $y = \sqrt{x}$ at $(1, 1)$
  2. $y = \sqrt{x}$ at $(4, 2)$
  3. $y = x^3$ at $x = -1$
  4. $y = x^3$ at $(0, 0)$
  5. $y = \sin x$ at $(0, 0)$
  6. $y = \sin x$ at $x = \frac{\pi}{2}$

Activity 14.3 Non Differentiability - Geometrical Meaning

Procedure:

- In the above applet,using input box, set $f(x) = |\sin x|$ (give input `abs(sin(x))`) and set a=0



What happens to the secant lines passing through the point (0,0) as $h \to 0$

What about the existence of the limit at a=0 ?

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

What about the existence of the tangent at (0,0)?

Discuss the existence of the tangents to the curves and the derivatives of the functions at the given points.

1. $y = \sin x \quad \text{at } x = \pi$
2. $y = \sin |x| \quad \text{at } x = \pi$
3. $y = |x| \quad \text{at } x = 0$
4. $y = \begin{cases} x^2 & \text{if } x \leq 2 \\ 2x & \text{if } x > 2 \end{cases} \quad \text{at } x = 2$
5. $y = x^{\frac{1}{3}} \quad \text{at } x = 0$
6. $y = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 6 & \text{if } x = 2 \end{cases} \quad \text{at } x = 2$
7. $y = \begin{cases} x^2 & \text{if } x \leq 2 \\ (x-4)^2 & \text{if } x > 2 \end{cases} \quad \text{at } x = 2$
8. $y = \begin{cases} x^2 & \text{if } x \leq 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases} \quad \text{at } x = 2$

Lab 15

Derivative of a function

Aim

- To explore the concept of the derivative of a function.

Concepts

- Concept of the derivative of a function at a point.
- Derivative of a function at a point is the slope of the tangent to the corresponding curve at that point.
- Graph of a function and its derivative.

Discussion

We discuss the concept of derivative of a function as a extension of the concept of derivative of a function at a point.

We plot the graph of the derived function using the idea that the derivative of the function at a point is the slope of the tangent to the curve at that point.

We also familiarise input commands for finding derivatives.

Activity 15.1 Relation Between a Function and its Derivative

Procedure:

- Draw the graph of the function $f(x) = x^2$
- Create a slider **a** with increment 0.001
- Input the points $A(\mathbf{a}, 0)$ and $B(\mathbf{a}, f(\mathbf{a}))$
- Draw AB using `Segment` tool
- Draw the tangent to the curve at the point B
- Show the slope **m** of the line (using `Slope` tool, click on the line).

For different values of **a**, find the slope of the tangent (*ie* $f'(\mathbf{a})$)

Sl. No	**a**	$f'(\mathbf{a})$
1	1	
2	1.2	
3	-2	
4	-1.8	

What is the relation between **a**, and $f'(\mathbf{a})$?

Activity 15.2 Graph of Derived Function

Procedure:

- In the above applet, open Graphics 2
- Plot the point $C(a, m)$ (y coordinate gives $f'(a)$)
- Trace the point C and give animation to the slider **a**
- Observe the path of the point C.(Using `Locus` tool click on the slider **a** and then on the point C, we get the path of C. Or we can use the input command `Locus[C,a]` to get the path)

Find the equation of the path

Activity 15.3 Equation of Derived Function

Procedure:

- In the above applet create an input box for f

Change the function f. Observe the locus of C and try to find its equation

Sl. No	f	Equation of locus of C
1	$x^2 + 1$	
2	$5x^2$	
3	x^3	
4	$x^3 - 2$	
5	$\sin x$	
6	$\cos x$	

- You can check your answer by drawing the curve, which you consider as the equation of the locus, and check whether it coincides with the locus

Activity 15.4 Derivative using Command

Procedure:

- Open GeoGebra window, input the function $f(x) = x^2$
- In Graphics 2, give any one of the following inputs
  f' or `Derivative[f]` which gives $f'(x)$. We can see $f'(x)$ from Algebra view and its graph from Graphics 2
- Create an input box for f

Find the derivatives of the following functions

Sl. No	$f(x)$	$f'(x)$
1	x^5	
2	$\frac{1}{x}$	
3	$\sqrt{x}$	
4	$\frac{1}{x^3}$	
5	$\sin x$	
6	$\tan x$	

Additional Activities

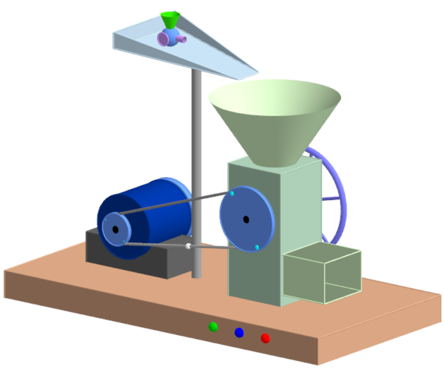
Activity 15.A Derivative Machine

Aim : We treat 'Derivative' as an operator, which changes one function to another. So we compare it with an input output machine. Here the inputs are functions. In Activity 1.3, we treated functions as machines. So Derivative Machine is a super machine which transforms a machine into another.

Procedure :

Use Applet ML 15.A

About the Applet :



This applet is similar to Applet ML 1.3 that we used in Activity 1.3.
Using the buttons provided on the machine we can operate it.
With the help of check boxes we can select a machine or functions as input.
We can change the input function with the help of an input box.

Using the applet find derivatives of some functions.

Lab 16

Miscellaneous

In this lab we do activities related to Complex numbers, Sequences and series and Linear inequalities.

Activity 16.1 Complex numbers

Aim

- To explore the geometry of modulus, argument and polar form of a complex number.

Concepts

- Modulus and argument of a complex number.
- Representation of a complex number on the Argand plane.
- Polar form of a complex number

Discussion

We plot a complex number on the Argand plane using GeoGebra and discuss modulus, argument and polar form geometrically.

Procedure

- Plot the complex number $z_1 = 1 + \sqrt{3}i$ (Input `1+sqrt(3)i`)

  Find its modulus and argument geometrically (without using input commands)

  Write the polar form of the complex number

- Create an input box for z_1.

  Find the polar forms of the following complex numbers

  1. $-1 - \sqrt{3}i$
  2. $\sqrt{3} + i$
  3. $3i$
  4. $-1 + i$
  5. -2
  6. $\dfrac{-8}{1 + \sqrt{3}i}$

- Plot the following complex numbers whose polar coordinates are given

  1. $(2, 40°)$
  2. $(5, -30°)$
  3. $(3, 90°)$
  4. $(2, 0°)$

Activity 16.2 Sequences and Series

Aim

- To generate sequences using GeoGebra commands.

Concepts

- General term of a sequence.

Discussion

- In this lab we familiarise GeoGebra commands for generating sequences.

Procedure

If we know the general term of a sequence, we can generate it using sequence command
For example,
`Sequence[n^2,n,1,10]` gives the first 10 terms of the sequence of squares of natural numbers. We can see it in Algebra view as a list.

- Create an integer slider **m**. The command `Sequence[3n+1,n,1,m]` gives the first m terms of the AP 4,7,10, $\cdots$

Find the general term and generate the first m terms of the following sequences.

1. 6, 10, 14, $\cdots$
2. 2, 4, 8, $\cdots$
3. $1, \frac{1}{2}, \frac{1}{4}, \cdots$
4. 0.1, 0.01, 0.001, $\cdots$
5. $\frac{1}{9}, \frac{-1}{27}, \frac{1}{81}, \frac{-1}{243}, \cdots$

Activity 16.3 Sum to n terms

Aim

- To find sum of sequences.

Concepts

- General term of a sequence.

Discussion

- Here we use GeoGebra commands to find the sum to a required number of terms of sequences and series. We also discuss the sum to infinity of a Geometric Progression.

Procedure

Using `Sum` command, we can find the sum of the elements of a sequence
For example,create the sequence of m natural numbers (say list1)

- `Sum[list1]` gives the sum of all elements of list 1

- `Sum[list1,5]` gives the sum of the first 5 elements of list 1
- `Sum[Sequence[n^2,n,5,10]]` gives the sum $5^2 + 6^2 + \cdots + 10^2$

Create a number slider **m**. Find the sum to **m** terms of the following sequences. What happens to the sum as the value of **m** increases?

1. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \cdots$
2. $\frac{1}{1\times 2}, \frac{1}{2\times 3}, \frac{1}{3\times 4}, \cdots$
3. $3^{\frac{1}{2}}, 3^{\frac{1}{4}}, 3^{\frac{1}{8}}, \cdots$
4. $2^2, \left(\frac{3}{2}\right)^2, \left(\frac{4}{3}\right)^3, \left(\frac{5}{4}\right)^4, \cdots$

Activity 16.4 Graphical Solution of Linear Inequalities

Aim

- To solve system of linear inequalities graphically.

Concepts

- Half - Plane
- Graphical solution of a system of linear inequalities

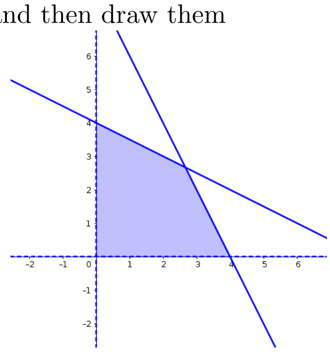
Discussion

- Using GeoGebra commands we can draw the regions represented by linear inequalities. In this lab we construct the solution region of a system of linear inequalities with the help of GeoGebra.

Procedure

- Imagine the half planes determined by the following inequalities and then draw them

  1. $x \leq 2$ [Use input command `x<=2`]
  2. $y \geq -3$
  3. $x < y$
  4. $2x + 3y \leq 6$
  5. $4x - 3y > 24$ [Use input command `4x-3y>24`]



- Draw each half plane given in the following system of inequalities and identify the common region representing the solution of the system

  $x + 2y \leq 8$
  $2x + y \leq 8$
  $x \geq 0$, $y \geq 0$

- We can draw the region representing the solution of the system using the command

  `x+2y<=8 && 2x+y<=8 && x>=0 && y>=0`

  OR

  x+2y $\leq 8 \wedge$ 2x+y$\leq 8 \wedge$x$\geq 0 \wedge$ y≥ 0

- Draw the solution region of the following system of linear inequalities.
  1. $5x + y \leq 20, x \geq 1, y \geq 2$
  2. $x - 2y \leq 3, 3x + y \geq 12, x \geq 0, y \geq 1$
  3. $x + 2y \leq 10, x + y \geq 1, x - y \leq 0, x \geq 0, y \geq 0$

Additional Activities

Activity 16.A Sum of Complex Numbers

Procedure:

Input commands
abs(z_1) gives the modulus of the complex number z_1.
Conjugate(z) gives $\overline{z}$

- We can plot a complex number using input command For example, the input `2+3i` gives the complex number $2 + 3i$
- Plot two complex numbers z_1 and z_2
- Using input command `z_1 + z_2` find their sum.
- Complete the quadrilateral whose vertices are z_1, z_2, $z_1 + z_2$ and the origin.

  What is the peculiarity of this quadrilateral?
- Change the positions of z_1 and z_2 and confirm your observation.

Activity 16.B Product of Complex Numbers

Procedure:

- Plot two complex numbers z_1 and z_2
- Plot the product of complex numbers z_1 and z_2 (say z_3), using the input command `z_1 * z_2`
- Show the modulus and argument of this complex number
- Create input boxes for z_1 and z_2

  Is there any relation between their arguments? Change z_1 and z_2 and confirm your observations

  Is there any relation between their amplitudes?

  If $z_2 = i$, what happens to their product?

  If $z_2 = -i$, what happens?
- Plot the complex number $\frac{z_1}{z_2}$ using the input command `z_1 / z_2`

  Find the relation between their amplitudes and arguments

Activity 16.C Square Root of a Complex Number

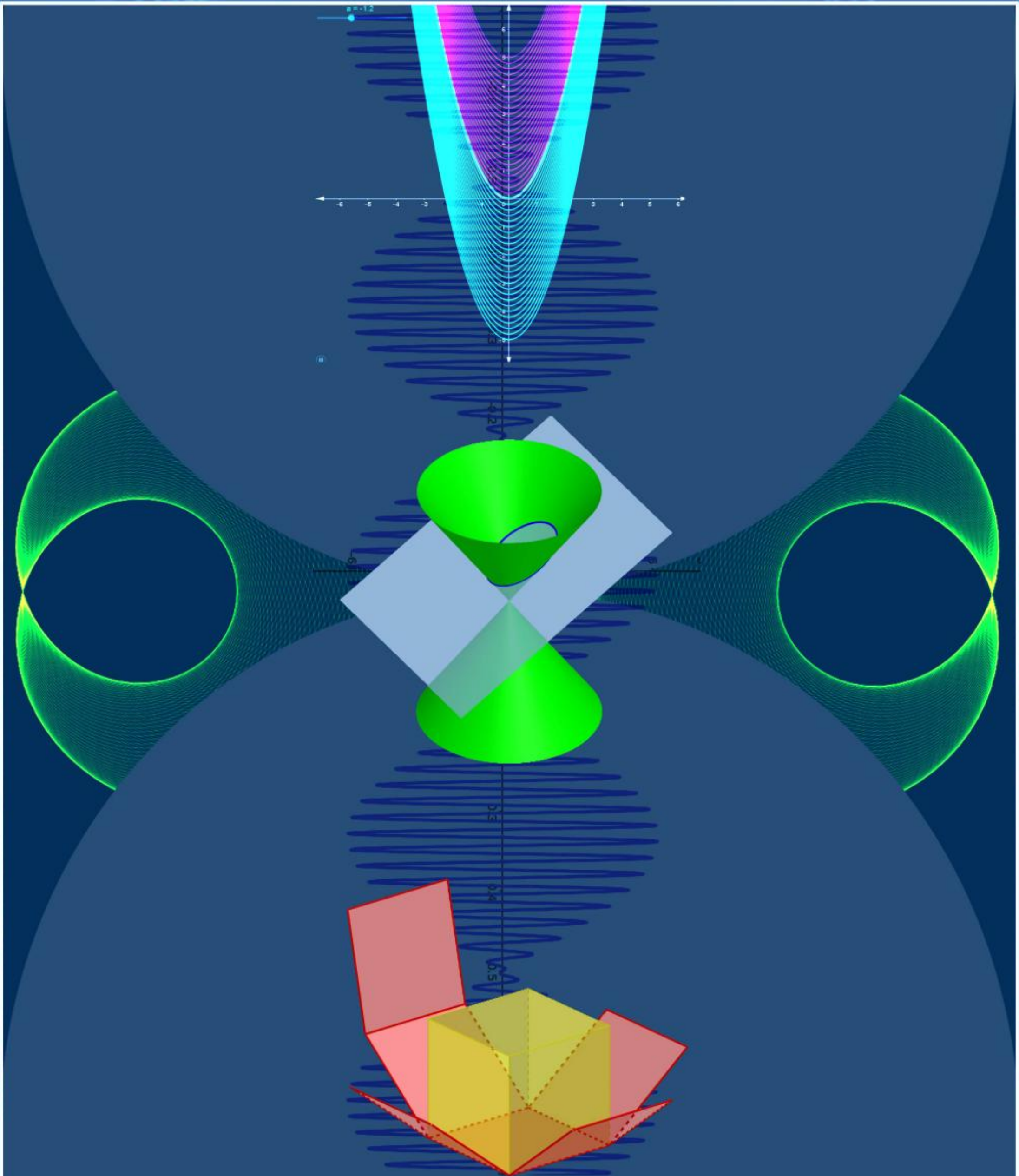
Procedure:

Instructions to find the square root of a complex number

- Create a number slider **r** with minimum value 0
- Create an angle slider θ
- Plot the complex number $z_1 = \mathbf{r}(\cos\theta + i\sin\theta)$
  [Input: `r(cos(`θ`)+i*sin(`θ`))`]
- Plot the square root of z_1 (say z_2)
  [Input: `sqrt(z_1)`]
- Join z_1 and z_2 with the origin and show their arguments and modulus

  Is there any relation between the arguments of z_1 and z_2 ?

  What is the relation between $|z_1|$ and $|z_2|$?
  Change the values of **r** and θ and confirm your observations.



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